Quality Provision in the Presence of a Biased Intermediary*

Alexandre de Cornière† and Greg Taylor‡

September 29, 2014

Abstract

In many industries, consumers rely on recommendations by an intermediary when choosing between competing products. In this paper, we look at how the existence of contracts between firms and intermediaries affects the quality of the advice received by consumers, and firms' incentives to invest in improving the quality of their products. We consider a model with one intermediary and two firms who decide how much to invest. Under a variety of contractual environments (vertical integration, ex post endorsement) we show that, even though the intermediary tends to endorse the best firm, contractual endorsement distorts firms' incentives to invest. Quality can then decrease or increase compared to an objective benchmark. We contrast our approach to a setup with fixed qualities and endogenous prices, under which contractual endorsement hurts consumers.

Keywords: intermediary, quality, bias.

JEL Classification: L1, L4, L86.

*We are grateful to Jacques Crémer, Mikhail Drugov, Anthony Dukes, Péter Esö, Renaud Foucart, Ben Hermalin, Paul Klemperer, Meg Meyer, Martin Obradovitz, Sander Onderstal, Andrew Rhodes, Chengsi Wang, Chris Wilson, and Peyton Young; seminar participants at Oxford and Toulouse; and participants at the 7th Télécom ParisTech ICT conference (2014), the 12th ZEW conference on the Economics of ICT (Mannheim, 2014), the CPB conference on Internet Economics and Privacy (The Hague, 2014), the 7th Conference on the Economics of Advertising and Marketing (Vienna, 2014), and the Summer Institute in Competitive Strategy (Berkeley, 2014) for useful comments and discussions. The authors gratefully acknowledge the financial support of this research by the NET Institute, http://www.NETinst.org.

†Department of Economics and Nuffield College, University of Oxford; adecorniere@gmail.com; https://sites.google.com/site/adecorniere

‡Oxford Internet Institute, University of Oxford; greg.taylor@oii.ox.ac.uk; http://www.greg-taylor.co.uk

1 Introduction

In many markets consumers rely on recommendations from an intermediary or other third party when choosing between competing products or services. For example, search engines provide a ranked list of the most relevant websites for a given query, physicians prescribe the best remedy for a given set of symptoms, and financial advisers are asked to recommend the best investments to their clients. While these are examples of situations in which intermediaries explicitly make recommendations, intermediaries may also make tacit suggestions by providing a default choice—such as the default search engine provided with a web browser, or the default web browser installed with an operating system.

Intermediaries have sought ways to profit from their position of influence. One way to do so is to set up a market (or negotiation) in which rival firms compete to be recommended.[1] A second common arrangement is for the intermediary to vertically integrate into the downstream market and recommend its own products—thereby capturing the associated downstream revenue.[2] The existence of these kinds of financial incentives raises questions about intermediaries' objectivity and, indeed, has attracted the attention of regulators. High profile interventions motivated by such concerns include recent FTC and European Commission investigations into Google's alleged search engine bias, a 2013 prohibition on the payment of commissions to financial advisers in the UK, and a 2010 case which saw pharmaceutical firm Novartis accused of paying illegal kickbacks to doctors.

The regulatory picture, though, is complex. Intermediaries serve a variety of roles (including helping consumers to find the best prices and identifying a good match with consumers' preferences), not all of which are well-understood. In this paper we focus on the role that intermediaries frequently play in helping consumers to find products of high quality—a role that often occupies a position of central importance in policy debates. One particular concern of competition authorities has been the potential for intermediary bias to reduce firms' incentives to invest in quality provision. Indeed, this dimension of the problem was at the center of the European Commission's case against Google (where parties reached a settlement in 2014).[3] A typical response to concerns that bias may diminish the incentive

[1] For example, the technology press reports that heavy bidding forced Google to pay $300 million for the right to be the default search engine in Mozilla Firefox (All Things D, 2011) and $1 billion for similar rights across Apple's suite of products (Recode, 2014). Other examples include doctors, who receive payments from pharmaceutical companies that are known to influence their prescription behavior (Engelberg, Parsons, and Tefft, 2013), and commissions paid by financial service providers to financial advisers.

[2] For example, Google is vertically integrated with a number of specialised websites (such as Google Maps and YouTube) and affords them favourable placement in its search results. Edelman and Lai (2013) have shown that this favouritism drives more traffic to these websites than they would enjoy without the upstream support.

[3] In the words of EU's competition commissioner J. Almunia (European Comission, 2014, our emphasis): "I believe that the new proposal obtained from Google after long and difficult talks can now address the

to invest is that, on the contrary, vertical restrictions provide scale efficiencies that promote rather than impede investment.

In this paper we seek to contribute to this debate by developing a formal model of intermediation with endogenous quality investment by downstream firms. Our setup allows us to study two potential sources of inefficiency in the market. The first is an *advice distortion*: do intermediaries have incentives to recommend the highest quality products to consumers, or do payments by downstream firms lead to collusive outcomes through which consumers end up using the wrong product? The second is an *investment distortion*: does a bias in an intermediary's recommendations cause firms to invest more or less in the quality of their product?

We consider a market composed of an intermediary, two firms, and a large number of consumers who can be either informed about firms' (endogenous) qualities or not. Uninformed consumers rely on the intermediary to choose among the two firms, whereas informed consumers directly go to the best firm. Investment in quality by a firm results in a higher utility for its consumers as well as in a higher per-consumer revenue for itself. In the baseline model, the intermediary cannot charge consumers for its recommendation.

We study three different contractual environments, and compare them to a benchmark in which the intermediary provides objective recommendations without charging firms. First, we characterize the intermediary's optimal contract: one firm contractually agrees not to invest, while the other invests as if it were a monopoly and receives the intermediary's endorsement. Next, we turn to *ex ante* endorsement contracts, under which firms compete to secure the endorsement *before* making their investment decision. We show that the firm that wins the endorsement has stronger incentives to subsequently invest than its rival, even though the ultimate quality may be lower than under the objective benchmark. Finally, we consider *ex post* endorsement contracts, where the decision of which firm to endorse is made *after* firms' investment decisions. The intermediary then ends up endorsing the best firm, but firms' investments are distorted downward due to a hold-up problem.

Comparing the various contractual environments, we therefore find that the intermediary usually endorses the best firm, i.e. that the quality of advice is not substantially distorted by endorsement contracts. This tendency for endorsements to favor high-quality products implies that it is rational for uninformed consumers to follow the recommendation of an intermediary even if they know it to be biased. Thus, our results do not depend upon consumer naivety and hold verbatim when consumers rationally decide whether to follow the intermediary's

Commission's concerns. Without preventing Google from improving its own services, it provides users with real choice between competing services presented in a comparable way; it is then up to them to choose the best alternative. *This way, both Google and its rivals will be able and encouraged to innovate and improve their offerings*."

advice or not. Though we find little reason to expect a distortion in advice, the same is not generally true for downstream firms' incentives to invest: these incentives can be stronger (for one firm) or weaker (for both firms) than under an objective benchmark.

Our main results are robust to a variety of extensions. First, if charging consumers is possible, and provided there are enough uninformed consumers, ex ante endorsement contracts allow the intermediary to charge higher consumer fees than would a commitment to provide objective recommendations. This is because ex ante endorsement induces an asymmetry in firms' qualities, making the intermediary's advice more valuable to consumers. It follows that neither the ability to extract consumers' surplus, nor a prohibition on payments from firms to the intermediary is sufficient to rule out undesirable bias. Our results also hold when we introduce competition on the intermediary market, explicit consumer search behavior or horizontal differentiation.

We also compare our results to those that stem from a model in which firms' primary strategic variable is their price.[4] Here we find a stark contrast: whilst endorsement of a quality-setting firm tends to lead to the best firm being endorsed, endorsement with price-setting generally leads to the intermediary endorsing the worst (i.e. highest-priced) firm. Moreover, endorsement of quality-setting firms can lead to an increase in quality that benefits consumers, whereas a endorsement under price-setting leads to higher prices.

Related literature

Our paper contributes to the recent literature on financial incentives for information intermediaries. Several papers consider models in which sellers compete to be recommended by an intermediary by offering commissions. Armstrong and Zhou (2011) show that such practices lead to mis-selling when sellers can monitor the intermediary's sales efforts, while Inderst and Ottaviani (2012a) study the effects of mandatory disclosure of commissions. In a search model, Hagiu and Jullien (2011) show that "diverting search" (i.e. providing sub-optimal advice) can be profit-maximizing even when commissions are symmetric, as it increases the amount of search by consumers and leads firms to charge lower prices. Federico and Régibeau (2011) show that a firm with a greater product variety has an inherent advantage in bidding for an intermediary's favour; nevertheless, the intermediary bias reduces welfare thanks to the consequent distortion in pricing. Inderst and Ottaviani (2012b) and Inderst and Ottaviani (2012c) discuss applications of this framework to the financial service industry, and consider issues related to consumer naivety and moral hazard for the intermediary. Our main contribution with respect to this literature is to endogenize the quality of sellers' products

[4] or any variable affecting consumers and firms in opposite directions.

while the above papers focus on their prices. As we show in section 8, the two approaches yield starkly differing conclusions as to the effects of endorsement contracts. Omitting investment decisions from the analysis can therefore lead to an overly pessimistic view of the role of endorsements.

Papers including Biglaiser (1993) and Lizzeri (1999) study intermediaries' role as certifiers of quality under the assumption that quality is exogenous. Durbin and Iyer (2009) and Peyrache and Quesada (2011) model the intermediary's incentives to accept bribes in this context. In a related vein, Biglaiser and Friedman (1994) and Dellarocas (2005) study endogenous quality, but focus on reputation as a mechanism to punish deviations from promised quality. We abstract from such reputational considerations and focus on the implications of endorsement contracting regimes for quality investment and advice. Buehler and Schuett (forthcoming) study how certification and minimum quality standards affect investment. Their setup differ from ours in that certification provides hard information, and that they do not consider (exclusive) contracts between the certifying authority and firms.

Our model is reminiscent of the literature on bundling and foreclosure in the sense that the intermediary can steer consumers towards its own product (see Rey and Tirole, 2007, for an overview, Choi, 2004, for a model with investment). Unlike these papers, in our model consumers do not derive intrinsic utility from the upstream product (the intermediary's recommendation), and bundling is "soft" in the sense that (informed) consumers can bypass the intermediary.

Some papers study related themes in the context of specific applications. Rutt (2011) and Jeon and Nasr (2012) study news aggregators that help readers identify high-quality news content, but do not study the possibility for firms to pay to get a better treatment from the intermediary (the aggregator). Choi and Kim (2010) and Economides and Hermalin (2012) study payments from content providers to Internet Service Providers in exchange for priority treatment. A key question, different from ours, is how such contracts affect the *intermediary*'s incentives to invest.[5] In both papers, unlike here, congestion of the network plays an important role. de Cornière and Taylor (forthcoming) consider the effects of integration or revenue-sharing agreements between a search engine and a content provider. There is no quality investment in that model. Instead, content providers' quality depends (negatively) on the quantity of advertising they display, and this quantity indirectly affects the search engine's advertising revenues.

One feature of our model is that it is rational for consumers to follow the intermediary's advice in spite of a known bias. In this respect, our paper is related to a literature (e.g.,

[5] Choi and Kim (2010) also consider investment incentives of the downstream firms (the content providers), but their modelling differs from ours in that investments do not benefit consumers.

Armstrong, Vickers, and Zhou, 2009) on environments with ordered search. Results in that literature depend upon consumers searching over prices. By contrast, we identify a novel mechanism, unique to quality provision, that results in the best firms being promoted irrespective of whether consumers actively search or not. Athey and Ellison (2011) and Chen and He (2011) endogenize the competitive process by which firms become prominent. Unlike these papers, we study firms' investment in quality, allowing us to study how biased recommendations affect investment incentives.

Even though we characterize the optimal contract (Proposition 1), the main results of our baseline model (Propositions 2 and 3) concern situations of incomplete contracting. For instance, ex ante endorsement bears a strong resemblance with vertical integration under incomplete contracting.[6] In different setups, Bolton and Whinston (1993) and Cremer and De Donder (2013) both study how the upstream firm's incentives to invest change under various market structures (e.g. integration or separation).

2 Model description

The market we consider is composed of two ex ante symmetric downstream firms, $i \in \{1, 2\}$, one intermediary, and a mass one of consumers who have homogenous preferences over the products.[7]

Firms Firm i's only strategic decision is a choice of quality, which determines (i) how much utility u_i product i delivers to each consumer, and (ii) how much revenue r_i firm i is able to generate per consumer.

We assume that both u_i and r_i are increasing in the quality of firm i.[8] The fixed cost of producing a quality that achieves a per-consumer revenue of r_i is $K(r_i)$, which we assume convex and such that $K(0) = K'(0) = 0$. Slightly abusing language, we will refer to r_i as the quality of firm i, and note $u_i = u(r_i)$.

Consumers There are two types of consumers in the market. *Informed* consumers can observe u_1 and u_2 prior to choosing a firm, and therefore always go to the firm with the highest

[6]Our market structure differs from standard models of vertical integration in the sense that the upstream firm (the intermediary) provides consumers, not inputs, to downstream firms. Also, unlike Grossman and Hart (1986), we ignore incentives problems within the firm.

[7]Our model is one of pure vertical differentiation. Most of our results carry over, with some nuances, in an environment with horizontal differentiation (see section 7.3 for an example).

[8]For instance, if downstream firms are free, ad-supported websites, an increase in quality benefits users and increases advertisers' willingness to pay. We provide more detailled examples in Section 6. We investigate the case in which $u'(r_i) < 0$ in section 8.2

quality. The mass of informed consumers is $1-\mu$. The remaining μ consumers are *uninformed*, and cannot observe the respective qualities before the match. However, uninformed consumers can consult the intermediary. In the baseline model consumers can only visit one downstream firm; we extend the model to allow for consumer search in Section 7.3.

The intermediary The role of the intermediary is to make a recommendation to uninformed consumers as to which firm to buy from. Our interest lies in situations in which the intermediary is, in large part, driven by financial incentives emanating from the seller side. In order to focus on this aspect, we begin with the assumption that the intermediary cannot receive any payment from consumers, and that consumer participation is inelastic. We study payments by consumers to the intermediary in section 7.1.

Contractual environment We study several contractual environments. The benchmark we use is a situation in which the intermediary is committed to recommending the best firm and receives no payment from firms. Even though this benchmark need not achieve the social optimum, it corresponds to the ideal of a disinterested and objective intermediary. We refer to this benchmark as the objective allocation rule. We next consider the optimal contract from the intermediary's point of view. This allows us to give an upper-bound on the latter's profit when the set of contracts is not restricted[9]—in particular, when quality choices are contractible upon. We next turn to simpler and more realistic environments by imposing further restrictions. In the *ex ante endorsement* model, the intermediary auctions its endorsement *before* qualities are chosen. This contractual environment captures some features of situations in which the decision by the intermediary to endorse a firm entails a greater commitment than the choice of quality by firms. For instance, when the intermediary owns or is owned by one of the firms, this firm chooses its quality knowing that uninformed consumers will be steered to it by the intermediary. In the *ex post endorsement* model, endorsement takes place *after* quality choices.

3 Benchmarks: objective intermediary and optimal contract

In most of the environments we consider, equilibria are in mixed strategies. In order to avoid introducing too much notation, we use F_i to denote the equilibrium distribution of r_i (F when the equilibrium is symmetric), and $\overline{r}$ to denote the upper-bound of the support. Note that in each section these objects take potentially different values, as the equilibria differ.

[9]Except that consumers cannot be charged.

3.1 Objective intermediary

We begin our analysis with the benchmark case in which the intermediary is exogenously constrained to accept no payment from firms and to recommend the firm with the highest quality.

Assuming symmetric tie-breaking, firm i's profits are

$$\pi_i = \begin{cases} r_i - K(r_i) \text{ if } r_i > r_j \\ r_i/2 - K(r_i) \text{ if } r_i = r_j \\ 0 \text{ if } r_i < r_j. \end{cases}$$

Since all consumers visit the highest quality firm in this benchmark, this game has the flavor of an all pay auction where investments are interpreted as bids (see Baye, Kovenock, and De Vries (1996) for instance). The novelty here is that, conditional on 'winning', both the payment and the value of the prize are determined by a firm's bid. There is a unique equilibrium in mixed strategies that can be characterised as follows:

Lemma 1. *The game with an objective inetrmediary has a unique equilibrium in which both firms set quality according to distribution $F(r) = K(r)/r$ on support $[0, \overline{r}]$, where $\overline{r}$ solves $K(r) = r$. Both firms earn zero (expected) profit in equilibrium.*

Proof. Claims 1–8 in Appendix A establish that any equilibrium must involve mixed strategies[10] on (symmetric) support $[0, \overline{r}]$, with both firms making zero profits. Thus, we have

$$\pi_i(r_i) = [\Pr(r_j < r_i) + \Pr(r_j = r_i)/2] r_i - K(r_i) = 0.$$

For this to hold at every $r_i \in [0, \overline{r}]$, it must be that $\Pr(r_j = r_i) = 0$ (by continuity). Writing $F_j(r_i) = \Pr(r_j \leq r_i)$:

$$F_j(r_i) r_i - K(r_i) = 0 \implies F(r) = K(r)/r.$$

To find $\overline{r}$, set $F(r) = 1$ and solve. ■

The fact that firms earn gross profits equal to zero and that the intermediary does not charge consumers implies that the intermediary's profits are also zero.

Since all consumers in this benchmark are directed to the highest quality firm with probability 1, there is no distortion in the advice process. However, there is a wasteful

[10]The intuition is fairly standard: Each firm would like to set its r slightly above that of its rival to capture all consumers (à la Bertrand). Once $r_i - K(r_i) = 0$, however, one of the duopolists must be making negative profits and prefer to set $r = 0$. At this point the rival firm would prefer to deviate to some lower, but positive r and the cycle starts again.

duplication of effort in the sense that both firms invest in quality provision; from a social standpoint, it would be possible to improve upon any equilibrium realisation of qualities by using a deterministic quality rule in which one firm's quality is set equal to zero.

3.2 Optimal contract for the intermediary

The industry profit (i.e. the sum of firms' and the intermediary's profits) is obtained when one firm serves all the consumers and chooses a quality $\widehat{r}$, with $K'(\widehat{r}) = 1$. Let $\widehat{\pi}$ be the industry profit in this case. Suppose that quality is both observable and verifiable by a court so that the intermediary can write (and enforce) essentially any contract contingent on realised firm qualities. The following result demonstrates that, using such a contract, the intermediary can extract the full monopoly industry profit, $\widehat{\pi}$.

Proposition 1. *If quality choices are observable and verifiable, there exists an equilibrium in which the intermediary's profit is $\widehat{\pi}$. Firm qualities are $r_1 = \widehat{r}$, $r_2 = 0$.*

Proof. The intermediary simultaneously offers two different contracts to firms 1 and 2. Let μ_i be the number of uninformed consumers that the intermediary sends to i. The contract for firm 1 stipulates a quality $r_1 = \hat{r}$, and a payment to the intermediary $T_1 = \widehat{\pi}$, and $\mu_1 = \mu$. The contract for firm 2 stipulates a quality $r_2 = 0$, a payment to the intermediary $T_2 = 0$, and $\mu_2 = 0$.[11] These offers are conditional on mutual acceptance: the intermediary commits to using the "objective" allocation rule $\mu_i = \mu \mathbf{1}_{\{r_i > r_j\}} + \frac{\mu}{2} \mathbf{1}_{\{r_i = r_j\}}$ without accepting any payment if one firm rejects its contract.[12]

Suppose that both firms accept. Then the μ uninformed consumers are sent to firm 1, and the $1 - \mu$ informed consumers also choose firm 1 because of its better quality. Firm 1's profit is then $\widehat{\pi} - \widehat{\pi} = 0$. Firm 2 also makes zero profit.

Can one firm profitably deviate by rejecting the contract and choosing some other r_i? The answer is no because, as we saw in the analysis of the "objective" case, both firms make zero profit.[13] ■

Interestingly, under the optimal contract, the intermediary is able to leverage its market power in order to extract the maximal industry profit, however small the share of uninformed consumers μ. The next section discusses some limitations of this result.

[11] Because qualities are observable and verifiable, if a firm accepts the contract but does not choose the right quality, it must pay an arbitrarily large fine.

[12] $\mathbf{1}_A$ is the indicator function for event A.

[13] This is not the unique equilibrium, as (reject,reject) is also an equilibrium. However by giving ϵ to each we can make sure that "accept" is a weakly dominating strategy.

4 Ex ante endorsement

The previous section demonstrates that a monopolist intermediary is able to achieve the maximal level of profit—even if the number of consumers, μ, it influences is small. However, two aspects of the optimal contract may be regarded as problematic. First, in the optimal contract, the intermediary signs a contract with firm 2, effectively ensuring that the latter stays out of the market under the threat of maximal competition. Second, the contract relies on quality being contractible. In particular, the intermediary must be able to punish a deviation from firm 2, who would otherwise be tempted to choose a quality $\hat{r} + \epsilon$.[14] In practice, it can be difficult to precisely measure firms' quality. Even if quality is observable it will often be non-verifiable—meaning that a third party such as a court cannot verify that a deviation from the contractually agreed quality has taken place. These considerations lead us to investigate two alternative contractual arrangements, the first of which we call *ex ante contracting*.

Under ex ante contracting, the intermediary enters into a contract with one of the firms *prior* to quality investment taking place. This contract stipulates that the intermediary will unconditionally recommend the counterparty's product in return for a transfer (determined by running a second price auction at the ex ante stage). Since performance in this contract does not depend upon quality, the non-verifiability of quality is no longer problematic. Formally, the timing of ex ante contracting is as follows:

1. The intermediary runs a second price auction in which the firms bid to win the intermediary's endorsement.

2. Both firms observe the outcome of the auction and simultaneously choose a level of investment r_i.

3. The intermediary reveals to uninformed consumers which firm it has endorsed.

4. Informed consumers visit the higher-quality firm, and uninformed consumers visit the firm endorsed by the intermediary.

Since the firms are ex ante symmetric, they both have the same willingness to pay for endorsement (given by the expected difference between profit with- and without endorsement). We adopt the convention that it is firm 1 that is recommended by the intermediary.

[14]Such a deviation is profitable only when μ is small enough, see Lemma 2.

Firms' profits are

$$\pi_1 = \begin{cases} r_1 - K(r_1) \text{ if } r_1 > r_2 \\ r_1 [\mu + (1-\mu)/2] - K(r_1) \text{ if } r_1 = r_2 \\ r_1\mu - K(r_1) \text{ if } r_1 < r_2, \end{cases}$$

$$\pi_2 = \begin{cases} r_2(1-\mu) - K(r_2) \text{ if } r_2 > r_1 \\ r_2(1-\mu)/2 - K(r_2) \text{ if } r_2 = r_1 \\ -K(r_2) \text{ if } r_2 < r_1. \end{cases}$$

4.1 Many uninformed consumers

It is instructive to begin with the case in which the intermediary serves as a gatekeeper for a relatively large proportion of consumers.

Lemma 2. *If $\mu \geq 1 - [K(\widehat{r})/\widehat{r}]$ then the unique equilibrium quality profile of the ex ante contracting game is in pure strategies and has $r_1 = \widehat{r}$, $r_2 = 0$. Profits gross of the payments to the intermediary are $\pi_1 = \widehat{\pi}$, $\pi_2 = 0$.*

Proof. Let us first show that the proposed strategy profile is an equilibrium of the subgame in which the intermediary endorses firm 1. Here, firm 1 achieves the monopoly profit, so it can obviously not do better. The only way for firm 2 to attract some consumers is to choose $r_2 > \widehat{r}$, in which case the informed consumers would choose it over firm 1. The best such deviation is $r_2 = \widehat{r} + \epsilon$ with ϵ close to zero. Indeed, the function $r \mapsto (1-\mu)r - K(r)$ is concave and its maximum is attained for $r < \widehat{r}$ (as $\widehat{r}$ maximizes $r \mapsto r - K(r)$). Firm 2's maximal deviation payoff is thus $(1-\mu)\widehat{r} - K(\widehat{r})$, which is non positive because $\mu \geq 1 - [K(\widehat{r})/\widehat{r}]$: there aren't enough informed consumers to make this deviation profitable.

To show uniqueness, note first that any $r_2 \in (0, r_1]$ is dominated by $r_2 = 0$. Therefore the only other possible equilibria in pure strategies must be such that $r_1 < r_2$. If $r_2 < \widehat{r}$, firm 1 has a profitable deviation: choose $r_1 = \widehat{r}$ and realize the monopoly profit. If $r_2 > \widehat{r}$, by the same argument of concavity of $r \mapsto (1-\mu)r - K(r)$, firm 2 could increase its profit by reducing r_2. The proof that there are no equilibria in mixed strategies can be found in Remark 1 and Claim 9 in Appendix A. ■

During the endorsement auction, firms' willingness to pay is given by the increase in gross profits associated with endorsement: $\widehat{\pi} - 0 = \widehat{\pi}$. Comparison of this result with Section 3.2 reveals that, provided the intermediary controls enough of the market, it is able to implement the optimal contract outcome via ex ante contracting—even if the optimal contract itself is non-enforceable.

Corollary 1. *If* $\mu > 1 - [K(\widehat{r})/\widehat{r}]$ *then ex ante contracting allows the intermediary to implement the payoffs associated with the optimal contract.*

The threshold on μ for Corollary 1 to hold need not be very large. For example, if costs have the form $K(r) = r^a$ then we have $\widehat{r} = (1-a)^{1/(a-1)}$ so that Corollary 1 holds for any $\mu > (a-1)/a$. It is clear that for costs that are approximately linear (as $a \to 1$), ex ante contracting implements the optimal scheme for any positive μ.

4.2 Many informed consumers

When the intermediary controls access to a smaller proportion of consumers, the equilibrium described in Lemma 2 breaks down (there are enough informed consumers for 2 to find competing in quality worthwhile). Equilibrium quality provision is then as described in the following result.

Lemma 3. *If* $\mu < 1 - [K(\widehat{r})/\widehat{r}]$ *then the unique equilibrium quality profile of the ex ante contracting game is as follows: Firm 1 randomizes over* $[\underline{r}, \overline{r}]$ *(with* $\underline{r} > 0$ *defined below) according to the cdf*

$$F_1(r) = \frac{K(r)}{(1-\mu)r}. \tag{1}$$

Firm 2 randomizes over $\{0\} \cup [\underline{r}, \overline{r}]$ *with*

$$F_2(r) = \frac{K(r) + \mu(\overline{r} - r)}{(1-\mu)r} \tag{2}$$

for $r \in [\underline{r}, \overline{r}]$ *and* $F_2(0) = F_2(\underline{r}) > 0$. *Profits are* $\pi_1 = \mu\overline{r}$, $\pi_2 = 0$.

Proof. All Claims referenced in this proof may be found in Appendix A. Claims 9–19 establish that any equilibrium must be such that the support of firm 1's strategy is $S(F_1) = [\underline{r}, \overline{r}]$, that $S(F_2) = \{0\} \cup [\underline{r}, \overline{r}]$, and that the only qualities that are played with positive probability are 0 (by firm 2) and $\underline{r}$ by firm 1. Firm 1 must be indifferent between any pair of qualities over its support. In particular, $\overline{r}$ must generate the same profit as any $r \in [\underline{r}, \overline{r})$. This gives us $\overline{r} - K(\overline{r}) = (\mu + (1-\mu)F_2(r))r - K(r)$. Using Claim 15 to replace $\overline{r} - K(\overline{r})$ by $\mu\overline{r}$ (through firm 2's zero profit condition), and rearranging the terms gives us (2). Expression (1) is obtained similarly.

To be complete, our characterization of the equilibrium must also include the determination of $\underline{r}$ and $F_2(0)$. The indifference condition for firm 1 between $\underline{r}$ and $\overline{r}$ is $\overline{r} - K(\overline{r}) = [\mu + (1-\mu)F_2(0)]\underline{r} - K(\underline{r})$. Using $K(\overline{r}) = (1-\mu)\overline{r}$ (by Claim 15), the previous equation can be rewritten

$$\mu\overline{r} = [\mu + (1-\mu)F_2(0)]\underline{r} - K(\underline{r}). \tag{3}$$

Substituting $K'(\underline{r})$ for $\mu + (1-\mu)F_2(0)$ (thanks to Claim 12) into (3) yields

$$\mu\overline{r} = \underline{r}K'(\underline{r}) - K(\underline{r}) \tag{4}$$

and $F_2(0)$ is obtained by rearranging (3) and using (4):

$$F_2(0) = \frac{K'(\underline{r}) - \mu}{1-\mu}. \tag{5}$$

Using claim 15, firm 1's profits are $\overline{r} - K(\overline{r}) = \overline{r} - (1-\mu)\overline{r} = \mu\overline{r}$. ■

Since the marginal benefit of winning the intermediary's endorsement is $\pi_1 - \pi_2 = \mu\overline{r}$, both firms bid up to $\mu\overline{r}$ for endorsement at the ex ante stage and this expression gives the intermediary's profit.

We can now state the main result of this section.

Proposition 2. *Under ex ante contracting:*

1. *Firm 1's quality first order stochastically dominates firm 2's quality,*

2. *Firm 2's quality falls compared to when the intermediary is objective (in the sense of first order stochastic dominance),*

3. *The distribution of firm 1's quality rotates compared to when the intermediary is objective, so that r_1 becomes less dispersed.*

Proof. See Appendix A. ■

The fact that strategies are stochastic implies that the firm recommended by the intermediary will sometimes be that with the lowest quality. This notwithstanding, the endorsed firm's quality first order stochastically dominates its rival's, implying that the intermediary's recommendations are, on average, in line with the interests of consumers. Consumers therefore find it rational to follow the intermediary's advice.

Figure 1 illustrates Proposition 2. It is interesting to consider the intuition behind it. Fix a distribution, F_1, for r_1. Heuristically, the marginal return to an increase in r_2 under an objective intermediary is

$$\frac{\partial \pi_2}{\partial r_2} = F_1(r_2) + F_1'(r_2)r_2 - K'(r_2).$$

There is a marginal effect (a higher quality increases expected demand) and an inframarginal effect (a higher quality implies more revenue per-consumer). The corresponding expression

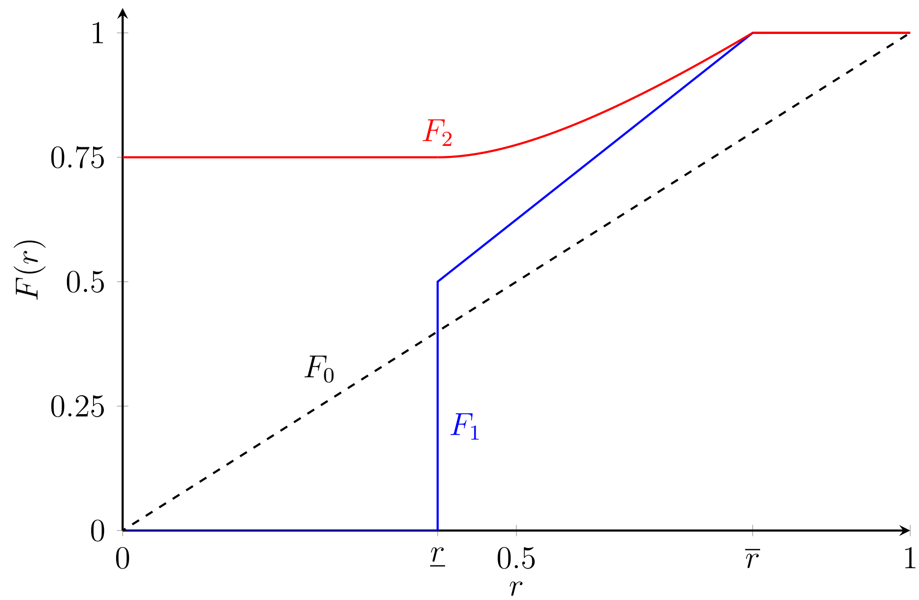

Figure 1: Equilibrium c.d.f. in the objective benchmark (F_0) and under ex ante endorsement (F_1 and F_2) with $K(r) = r^2$ and $\mu = 1/5$.

under ex ante contracting is

$$\frac{\partial \pi_2}{\partial r_2} = (1-\mu)F_1(r_2) + (1-\mu)F_1'(r_2)r_2 - K'(r_2) \, .$$

We see that endorsement affects the incentive to provide quality through two channels. The first, captured by $(1-\mu)F_1(r_2) < F_1(r_2)$, is a *scale effect*. Since costs are fixed, firms enjoy economies of scale in quality provision. By diverting consumers to firm 1, endorsement reduces firm 2's expected scale and thus its quality provision incentive. The second effect, reflected in $(1-\mu)F_1'(r_2)r_2 < F_1'(r_2)r_2$, is a *competition effect*. Under endorsement, the uninformed consumers' demand is insensitive to quality so that the total mass of consumers over whom the firms compete is reduced. As a consequence, a given increase in quality results in a smaller number of the rival firm's customers being poached so that the incentive to compete in quality is diminished. Both the scale and competition effects point in the same direction, resulting in a (stochastically) lower equilibrium quality for firm 2.

Fixing the distribution of r_2 at F_2 and repeating this exercise from firm 1's perspective, we see that the marginal profit from quality provision under an objective intermediary is

$$\frac{\partial \pi_1}{\partial r_1} = F_2(r_1) + F_2'(r_1)r_1 - K'(r_1) \, ,$$

whereas that under ex ante contracting is

$$\frac{\partial \pi_1}{\partial r_1} = [\mu + (1-\mu)F_2(r_1)] + (1-\mu)F_2'(r_1)r_1 - K'(r_1).$$

The competition effect experienced by firm 2 is also present here: like 2, firm 1 does not have to compete for consumers who will be directed to it anyway. However, $\mu + (1-\mu)F_2(r_1) > F_2(r_1)$ so the the direction of the scale effect is now reversed. By directing uninformed consumers to firm 1, the intermediary endows 1 with larger scale, which provides an impetus for 1 to increase its quality. Since the competition and scale effects work in opposite directions, the affect of ex ante contracting on firm 1's expected quality is, in general, ambiguous.

To get a handle on ex ante endorsement's overall effect on quality, suppose that costs have the constant-elasticity form $K(r) = r^a$ (with $a > 1$).[15] Under the objective benchmark, quality is distributed according to $F(r) = r^{a-1}$ and every consumer visits the best firm. The expected quality experienced by consumers is therefore

$$r^e = E(\max\{r_1, r_2\}|\text{objective}) = \int_0^1 2(a-1)r^{2(a-1)}dr = \frac{2a-2}{2a-1}.$$

Under ex ante contracting with $\mu > 1 - [K(\widehat{r})/\widehat{r}]$,[16] only firm 1 is active and its quality is given deterministically by $\widehat{r} = (1/a)^{1/(a-1)}$. Solving $E(\max\{r_1, r_2\}|\text{objective}) \leq \widehat{r}$ for a reveals that the objective benchmark results in higher quality if and only if $a > 1.37$. This example illustrates the role of returns to scale in determining which regime (competition under the objective benchmark, or monopoly under ex ante contracting) is superior. When returns to scale in quality provision are strongly decreasing (high a), it is better to rely on competition to generate investments because a monopolist would choose a low quality. The reverse holds with relatively large returns to scale (i.e. a close to, but above 1).

For smaller μ firm 2 becomes active under ex ante contracting and the average quality experienced by consumers is then

$$r^e = \mu E(r_1|\text{ex ante}) + (1-\mu)E(\max\{r_1, r_2\}|\text{ex ante}).$$

Figure 2 compares this average experienced quality to that which arises under the objective benchmark.[17] The above intuition continues to hold: objectivity improves consumers' experienced quality only when costs are sufficiently elastic with respect to quality (a is sufficiently

[15] $a = K'(r)r/K(r)$ is the elasticity of cost with respect to quality.

[16] Since, for $\mu > 1 - [K(\widehat{r})/\widehat{r}]$, Ex ante endorsement is equivalent to the optimal contract, the following discussion applies equally to either arrangement.

[17] If consumers are risk neutral (utility is linear in r) then the figure can be reinterpreted as showing when ex ante endorsement increases consumer surplus.

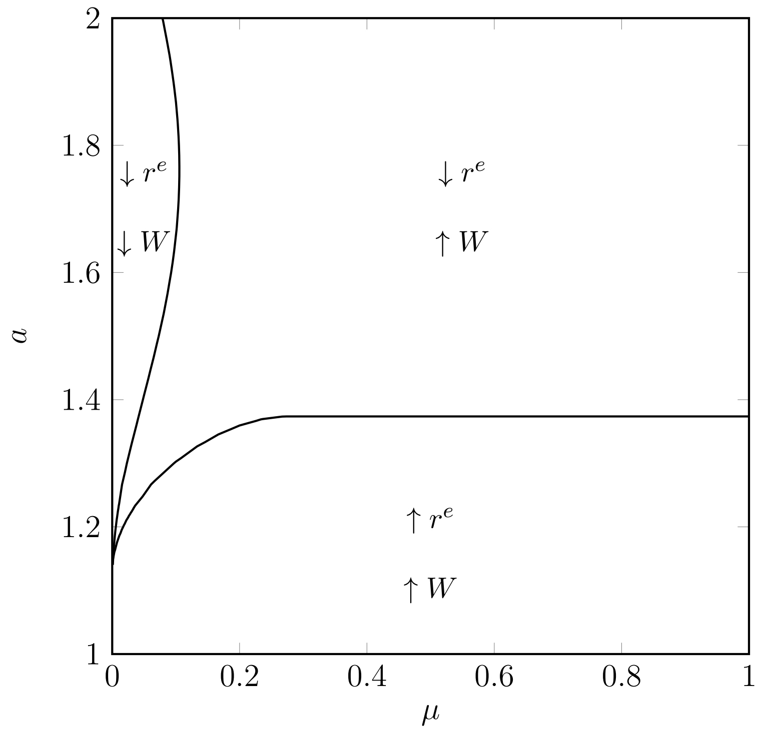

Figure 2: Effect upon welfare, W, and consumers' average experienced quality, r^e, of moving from the objective rule to ex ante endorsement.

large).

The comparison with standard contests (or all pay auctions) is interesting in this case. In standard models of one-shot contests between symmetric players under complete information, arbitrarily favoring one player decreases everyone's incentives to exert effort (Lazear and Rosen, 1981; Franke et al., 2013).[18] Here, on the other hand, because the reward conditional on winning depends on the level of effort, favoring firm 1 may increase its incentives to exert effort and result in a better quality for consumers.

Figure 2 also shows the effect of ex ante endorsement on total welfare under the assumption that consumers and firms symmetrically share the surplus from trade (i.e., $u(r) = r$).[19] Irrespective of whether ex ante endorsement increases quality or not, it delivers a given level of quality in a more efficient manner than does the objective benchmark. Intuitively, when the intermediary behaves objectively there is wasteful duplication of effort because the low-quality firm's product is never consumed. By distorting firm 2's quality downward,

[18] This is no longer necessarily true if players interact more than once, as shown in Meyer (1992), if there is incomplete information, or if bias is not additive or multiplicative (Drugov and Ryvkinz, 2013).

[19] Thus, welfare is given by $W = 2\mu E(r_1|\text{ex ante}) + 2(1-\mu)E(\max\{r_1, r_2\}|\text{ex ante}) - E(K(r_1)|\text{ex ante}) - E(K(r_2)|\text{ex ante})$.

ex ante endorsement mitigates this duplication of effort. Indeed, for $\mu > 1 - [K(\widehat{r})/\widehat{r}]$ it is eliminated altogether because firm 2 is completely foreclosed. Accordingly, Figure 2 shows that when μ is large (viz. when the downward distortion in 2's quality is large), ex ante endorsement is likely to be welfare-increasing.

5 Ex post endorsement

From a practical perspective, one attractive property of the ex ante contractual form is that it does not require quality to be observable or verifiable. Notice, however, that a consequence of writing a contract in which payments are not contingent upon quality is that some realisations of quality induce negative profits for the endorsed firm. In particular, if $r_2 > r_1$ then firm 1's gross profit is $\mu r_1 - K(r_1)$. Since firm 1 pays a fee of $\mu\bar{r}$ to the intermediary, its net profit is $\mu(r_1 - \bar{r}) - K(r_1) < 0$. In the presence of limited liability, the endorsed firm might then be unable to honor the payment to the intermediary.

One simple solution is to auction the endorsement *ex post* (i.e., after qualities are realised). Since bids can typically be observed and verified, this retains the attractive feature that quality need not be contractible. Unlike ex ante bidding, however, firms know their own quality at the time of the ex post auction; we can therefore be sure that they will never bid more than they are willing to pay. Ex post contracts also have other practical advantages. Many products involve lengthy, risk-prone development cycles that make negotiating at the ex ante stage problematic. Other markets, such as markets for online advertisements, are highly fluid on both the demand and supply side, [20] making shorter-term ex post contracts more practicable than the long-term commitment implied by ex ante contracting.

The timing is as follows: at $t = 1$, both firms simultaneously choose a level of investment r_1 and r_2. At $t = 2$, each firm submits a bid b_i. The highest bidder wins the endorsement, and pays the second-highest bid. At $t = 3$, informed consumers buy from the highest-quality firm, and uninformed consumers buy from the firm endorsed by the intermediary.

The second-price auction without reserve price is a convenient auction format to study, as truthful bidding is a weakly dominating strategy.[21] Given such truthful bidding, the winner of the auction is always the firm with the highest quality, r_i.

If $r_i > r_j$, firm i's profit is composed of two parts: it makes $(1 - \mu)r_i$ from informed consumers who do not consult the intermediary, and $\mu(r_i - r_j)$ from the uninformed consumers, as r_j is the price determined through the auction. Using arguments similar to those in the

[20]Indeed, much online advertising is sold in real-time auctions so that the shortest possible contracting horizon of a single consumer impression is used.

[21]But the results hold under different mechanisms, e.g. a proportional fee τr_i paid by firm i to the intermediary.

previous sections, one can show that the unique equilibrium is a symmetric equilibrium in mixed strategies, with support $[0, \overline{r}]$. Firm i's expected profit, if it chooses a quality r_i and expects firm j to play according to a distribution F, is thus

$$\pi_i(r_i) = \int_0^{r_i} ((1-\mu)r_i + \mu(r_i - r_j))\, dF(r_j) - K(r_i). \tag{6}$$

After an integration by parts and some simplifications, the previous expression becomes

$$\pi_i(r_i) = (1-\mu)r_i F(r_i) + \mu \int_0^{r_i} F(r_j) dr_j - K(r_i). \tag{7}$$

Over the support $[0, \overline{r}]$, profit is constant. Differentiating (7) with respect to r_i, we obtain the following first-order differential equation:

$$\frac{dF}{dr_i} + \frac{F(r_i)}{(1-\mu)r_i} = \frac{K'(r_i)}{(1-\mu)r_i}. \tag{8}$$

We then have

Lemma 4. *Under ex-post endorsement, qualities are drawn from a distribution F given by*

$$F(r) = \frac{\int_0^r \frac{K'(x)}{(1-\mu)x^{\mu}} dx}{r^{1-\mu}} \tag{9}$$

over a support $[0, \overline{r}]$, where $\overline{r}$ is given by $F(\overline{r}) = 1$.

Proof. The solution to (8) is $F(r_i) = \frac{\int_0^{r_i} \frac{K'(r)}{(1-\mu)r^{\mu}} dr + A}{r_i^{1-\mu}}$, where A is a constant. The initial condition $F(0) = 0$ then gives $A = 0$. ■

Example 1. *When the cost function K is quadratic, $K(r) = r^2$, we have $F(r) = \frac{2r}{(1-\mu)(2-\mu)}$, i.e. qualities are uniformly distributed over $[0, \frac{(1-\mu)(2-\mu)}{2}]$.*

We can now compare the distribution of qualities under ex-post endorsement with that under the objective benchmark.

Proposition 3. *Under ex post endorsement through a second-price auction, the distribution of quality is first-order stochastically dominated by that under the objective allocation rule.*

Proof. As in section 4, the equilibrium distribution of quality under the objective allocation rule is the same as that under ex post endorsement when $\mu = 0$:

$$F(r)|_{\mu=0} = \frac{\int_0^r K'(x)dx}{r} = \frac{K(r)}{r}$$

We also have

$$\frac{dF(r)}{d\mu} = \int_0^r \frac{K'(x)}{x^\mu r^{1-\mu}} \frac{1 + (1-\mu)(\log(r) - \log(x))}{(1-\mu)^2} dx \geq 0,$$

which proves the proposition. ■

Although the ex post mechanism efficiently selects firms for endorsement, some of the endorsed firm's revenue is extracted by the intermediary. This gives rise to a standard hold-up problem, the anticipation of which reduces *all* firms' incentives to invest. Quality is therefore stochastically lower than under objective recommendation.

Finally, we can compare the profit of the intermediary under ex ante and ex post endorsement.

Proposition 4. *Suppose that $K(r) = r^\alpha$, with $\alpha > 1$. Then the intermediary's profit is higher under ex ante endorsement than under ex post endorsement (with a second-price auction).*

Proof. To avoid ambiguity, let $\overline{r}_A$ and $\overline{r}_P$ be the upper bounds of the supports of the distribution under ex ante and ex post endorsement respectively. Under ex ante endorsement, the intermediary's profit π_A equals firm 1's profit, i.e. $\overline{r}_A - K(\overline{r}_A)$. We know that $\overline{r}_A$ is such that $(1-\mu)\overline{r}_A - K(\overline{r}_A) = 0$, which implies $\pi_A = \mu\overline{r}_A = \mu(1-\mu)^{\frac{1}{\alpha-1}}$. On the other hand, the profit under ex post endorsement is $\pi_P = \mu E[\min\{r_1, r_2\}|\text{ex post}]$. Solving for $F(\overline{r}_P) = 1$ using (9) with the functional form for K, we get $\overline{r}_P = \left(\frac{\alpha-\mu}{\alpha}\right)^{\frac{1}{\alpha-1}} (1-\mu)^{\frac{1}{\alpha-1}} = \left(\frac{\alpha-\mu}{\alpha}\right)^{\frac{1}{\alpha-1}} \overline{r}_A < \overline{r}_A$. We then have $\pi_P = \mu E[\min\{r_1, r_2\}|\text{ex post}] < \mu\overline{r}_P < \mu\overline{r}_A = \pi_A$. ■

6 Applications

Integration in the search engine industry As mentioned in the introduction, one of the motivating examples of the paper is the question of vertical integration in the search engine industry. Google has recently been accused of providing unfair preferential treatment to its own "downstream" websites , such as Google Maps or Youtube (see Edelman, 2014 for a description of the issue and a case against Google). Arguing that Google is an essential facility and that it shouldn't be in position to favor some content, especially if it has a financial interest in doing so, some observers have asked for the institution of a "right to access" that would guarantee a level playing field on the downstream market.

Our model can shed light on some of the issues at stake. Firms 1 and 2 are content websites, and the intermediary is a search engine. While some Internet users know their relative preferences between downstream websites and access them directly, the search engine plays a major role in directing "uninformed" consumers towards certain websites.

Let us check that the assumption that u and r are both increasing in quality makes sense in this setup. Suppose that content websites' revenues come exclusively from advertising. When a consumer visits a website of quality s, he must choose how long to stay on the website (i.e. how much of the content to consume). If he spends t units of time, his utility is $v(s,t) - c(t)$, where $\frac{\partial^2 v(s,t)}{\partial s \partial t} > 0$ and $c(t)$, the opportunity cost of time, is increasing. By revealed preference arguments, the time spent on a website of quality s, $t^*(s)$, is increasing in s. The website displays one advertisement, which the consumer notices between t and $t + dt$ with probability λdt. If advertisers' willingness to pay to be noticed by a consumer is a, the website's per-visitor revenue is then $r = \lambda a t^*(s)$. Consumer surplus, assuming they derive no utility from advertising, is then $u = v(s, t^*(s)) - c(t^*(s))$. Both r and u are indeed increasing in quality, hence we can apply our results.

We model integration between the search engine and a content website as an *ex ante* contract: when they choose their quality, integrated websites know that they will benefit from a prominent position. Our model predicts that integrated websites tend to invest more than their rivals (Proposition 2), and that integration is an effective way of foreclosing downstream rivals.

Recently, the European Commission obtained from Google an agreement to grant more exposure to some rivals of its downstream sites.[22] To a first-order approximation, such an agreement may be viewed, in our model, as a move towards the objective benchmark, i.e. a decrease in μ. Therefore our model predicts that the agreement will have a positive effect on Google's rivals' incentives to invest, in line with J. Almunia's statement quoted in footnote 3. However, our model does not justify the claim that Google's own incentives will also improve, as they can go both ways. In fact, our results show that, even though vertical integration between Google and a downstream website may foreclose competition, it can also be justified on the basis of efficiency, because it gives Google enough scale to induce extra investment.

Other applications to technology markets Beyond search engine integration, the technology industry offers other examples of applications for our framework. For instance, most web browsers' revenues come from the deals they have with the search engine they choose as the default option. In that case, the intermediary is the browser, and firms are the search engine companies. The $1 - \mu$ informed consumers are those who are savvy enough to change the default parameters, while the μ uninformed consumers have high switching costs. We can then reinterpret $t^*(s)$ as the number of queries a user makes to a search engine of quality s, and λ as the fraction of queries who result in the consumer clicking a sponsored link.

While *ex ante* contracts may be a fair representation of the relationship between Chrome

[22]Some of Google's opponents criticize the agreement (Fair Search, 2014).

and Google Search (or any integrated browser-search engine structure), some browsers, such as Mozilla Firefox or Apple's Safari, regularly let search engines compete for the right to be the default option. To the extent that search engines' past investments in quality are sunk at the time of negotiations with browsers, *ex post* contracting seems to better capture the essence of the relationship.

In the smartphone market, Google has recently been sued over claims it abused its market power by forcing hand-held device makers that use its Android operating system to also provide the search engine company's applications (such as Gmail, Google Maps or Youtube) as default options.[23] Most of these applications being free to use and ad-financed, our model can again be readily applied. Think of firms 1 and 2 as competing maps applications (Google Maps and MapQuest for instance). As above, μ is then the share of "captive" consumers, unaware of the existence of alternatives or having prohibitively high switching costs. A different interpretation could be that μ is the market share of manufacturers having signed a tying agreement with Google. Our model indicates that these (*ex ante*) agreements distort incentives to invest in asymmetric ways, which may or may not benefit consumers, depending on the elasticity of the cost function.[24]

Physicians and pharmaceutical companies Our model also has applications in non-technology markets, providing that firms' ability to attract consumers by lowering prices is limited (i.e. u_i and r_i must be positively correlated). Environments satisfying this condition include regulated markets or markets in which prices are imperfectly observed when the consumer chooses which firm to deal with.[25]

Consider for instance a situation in which physicians advise their patients as to which drug to buy. The following model is formally equivalent to ours. Suppose that there is a continuum of uninformed patients, a continuum of doctors, and two pharmaceutical companies offering competing treatments for a given condition. A share $1-\mu$ of doctors are honest: they are committed to offering the best available treatment, and they refuse payments by firms. The remaining μ doctors are corruptible: they accept payments in exchange for recommending a given treatment, even if it is inferior. The simplest way to model this is to assume that each corruptible doctor runs a second price auction and always recommends the winner's product to his patients.[26] Patients cannot observe whether a doctor is corruptible, and thus choose randomly which doctor to consult. Suppose that the utility, in monetary terms, of receiving a

[23] See Bloomberg (2014).

[24] Admittedly, the model neglects dynamic aspects of the question. Future research in that direction should be valuable.

[25] We discuss environments in which the condition does not hold in section 8

[26] Under more realistic and interesting scenarios corruptible doctors could for instance commit to recommend a given firm conditional on a minimum quality level.

treatment of quality s is s. Patients must pay a fixed price $\overline{p}$ to receive a prescription, while pharmaceutical companies and the bill payer (i.e., the insurer or public health body) negotiate a price $p(s)$, with $p' > 0$.[27] We then have $r = p(s)$ and $u = s - \overline{p}$, both again increasing functions of quality.

In this application, it seems that *ex post* contracts are a better description of the existing arrangements: pharmaceutical companies' efforts to lobby doctors presumably mostly occur for existing drugs, i.e. such that R&D efforts have already been sunk.

Our model then indicates that financial payments to doctors should be discouraged. While this result is not surprising, an interesting point here is that payments hurt consumers not because they induce doctors to recommend inferior drugs, but because they distort firms' initial incentives to invest in R& D.

Payola and other promotional practices Payola is the practice of bribing someone in return for the unofficial promotion of a product in the media. The most famous example is the music industry in the 1950's, where some influential disc-jockeys were prosecuted for failing to disclose the bribes they received from record companies. As Caves (2000) discusses, payola bears some strong resemblance with other practices (such as product placement in movies or prominent displaying of books by chain bookstores). Caves also mentions that in some cases, vertical integration serves the same purpose as payola: following the merger, ABC has a natural incentive to broadcast Disney movies instead of movies produced by other studios, and this will boost Disney's revenues from related products for instance.

Interpreting μ as the share of consumers who exclusively learn of new songs by listening to a specific station, $u(s_i)$ and $r(s_i)$ as respectively the expected utility and firm i's per-listener expected future revenue from album sales, both as functions of the album's quality s_i, it is relatively easy to see how our model can apply to these situations. Whether the above practices are best described as ex-ante or ex-post contracts depends on the specific circumstances of each case, and is probably subject to interpretation.

Our purpose in this section is to highlight the wide range of applications of our model rather than to provide a complete analysis of each case. Indeed the baseline model is a very stylized representation of the markets mentioned above, ignoring important aspects such as competition between intermediaries, payments by consumers to the intermediary, or horizontal differentiation between downstream firms. We now turn to these issues, and show that our main results are robust to these considerations.

[27] Berndt, 2002 and Lu and Comanor, 1998 report that more sophisticated drugs allow pharmaceutical firms to extract a larger unit price from the bill paying entity.

7 Extensions

7.1 Charging for advice

We have seen that endorsement contracts can result in a decrease in quality compared to the objective benchmark (e.g. Figure 2). While this is a cause for concern, one might argue that the result is driven by the intermediary's inability to extract consumers' surplus. Indeed, if the intermediary could commit to an allocation rule and charge consumers, it might be optimal to commit to the objective rule and extract consumers' surplus. The argument has some merits, but also some limitations. First, assuming that the intermediary can credibly commit with respect to consumers is a rather strong assumption. Second, even if one accepts this assumption, it does not follow that it is more profitable to implement the objective rule compared to, say, the ex ante contract. Consider the following extension: Suppose that in the first stage the intermediary can commit to a probability β with which each uninformed consumer is sent to the endorsed firm. With probability $1-\beta$, the consumer is sent to the best firm (which may or may not be the endorsed one). This formulation encompasses the objective allocation rule ($\beta = 0$) as well as the endorsement rule studied in earlier sections ($\beta = 1$). For simplicity, we assume that consumers get a payoff r_i if they visit firm i, which might come from symmetric Nash bargaining between firms and consumers.[28]

The timing is the following:

1. The intermediary chooses its level of "bias" β, and a price T that consumers must pay to use it.

2. Consumers observe β and T and decide whether to use the intermediary.

3. Firms bid for the endorsement.

4. Firms observe the outcome of the auction, and choose their level of investment r_i.

5. The consumers who use the intermediary are sent to the endorsed firm with probability β. The remaining users of the intermediary, as well as the $1-\mu$ informed consumers, visit the best firm. The uninformed consumers who do not use the intermediary visit one firm randomly.[29]

Given that consumers are homogeneous, the intermediary chooses a price T that makes consumers indifferent between using it and searching randomly. We assume that, when an

[28] Extending these results to a more general payoff $u(r_i)$ is straightforward.

[29] An alternative formulation is to assume that those consumers can leave the market and get a constant outside-option. The analysis of that case is available upon request.

equilibrium with full participation exists, consumers coordinate on it.[30]

For a given level of β, assuming that all uninformed consumers use the intermediary, the subgame starting at $t = 3$ is equivalent to the ex ante endorsement game with a number of uninformed consumers $\beta\mu$.

In order to determine the maximal T compatible with full participation for a given β, let us compute a consumer's willingness to pay for the advice of the intermediary. If the consumer visits a random firm, his expected utility is $\frac{E[r_1+r_2|\beta\mu]}{2}$. If he uses the intermediary, he will be directed to firm 1 with probability β, and to the best firm with the complementary probability. The maximal fee $T(\beta, \mu)$ that can be charged is therefore

$$T(\beta, \mu) = \beta E[r_1|\beta\mu] + (1-\beta)E[\max\{r_1, r_2\}|\beta\mu] - \frac{E[r_1 + r_2|\beta\mu]}{2}.$$

When the cost function is quadratic ($K(r) = r^2$), we have, for $\beta\mu < 1/2$:[31]

$$T(\beta, \mu) = \frac{\beta(1+\mu)-1}{2} + \frac{\beta\mu}{4}\ln\left(\frac{1-\beta\mu}{\beta\mu}\right) + \frac{1-\beta}{6(1-\beta\mu)}\left(4 + \beta\mu(4\beta\mu + 8\sqrt{\beta\mu(1-\beta\mu)} - 11)\right) \quad (10)$$

When $\beta\mu > 1/2$, only firm 1 invests a positive amount, and $T(\beta, \mu) = 1/4$.

Let $\beta_T(\mu)$ be the level of bias that maximizes uninformed consumers' willingness to pay for the intermediary's service when there are μ uninformed consumers. We have the following:

Proposition 5. *For quadratic K, there exists $\tilde{\mu} > 0$ such that $\mu > \tilde{\mu} \Rightarrow \beta_T(\mu) > 0$, i.e. the intermediary chooses to be biased in favor of one firm.*

Proof. Using (10), we have $\lim_{\beta\to 0} T(\beta, \mu) = 1/6$. We also have $T(1, \mu) = \frac{\mu}{4}(2 + log(\frac{1-\mu}{\mu}))$ for $\mu \leq 1/2$. $T(1, \mu)$ is increasing in μ over $\mu \in (0, 1/2)$, with $\lim_{\mu\to 0} T(1, \mu) = 0$ and $T(1, 1/2) = 1/4 = T(1, \mu)$ for $\mu > 1/2$. Thus there exists μ' such that $T(0, \mu) < T(1, \mu)$ for any $\mu > \mu'$. ■

One can check numerically that $\beta_T(\mu) = 1$ for $\mu > \mu^* \approx 1/5$. When μ is large, choosing $\beta = 1$ creates a large asymmetry in firms' incentives to invest. Consumers are reluctant to pick a firm randomly, because firm 2 is likely to be of poor quality, and the intermediary can thus charge a higher fee.

The condition $\mu > \mu^*$ is a sufficient one for ex ante endorsement to dominate. Indeed, we have only considered the profit that can be extracted from consumers. Because ex ante endorsement, unlike the objective allocation rule, allows intermediaries to extract a positive

[30] The coordination aspect of the problem arises because, as we shall see, the intermediary's advice is more valuable to a consumer as more consumers follow it.

[31] 1/2 corresponds to the threshold $1 - K(\hat{r})/\hat{r}$ from Lemma 3 when $K(r) = r^2$.

amount from firms, it will be preferred by the intermediary even for values of μ slightly below μ^*.

When $\beta = 1$, the expected quality experienced by consumers is

$$\mu E[r_1|\beta = 1] + (1-\mu)E[max\{r_1, r_2\}|\beta = 1] = \frac{2}{3}(1 + \mu^2 - 2\mu(1 - \sqrt{(1-\mu)\mu}))$$

if $\mu < 1/2$, and is equal to $1/2$ if $\mu \geq 1/2$. When $\beta = 0$, the expected quality is $E[max\{r_1, r_2\}|\beta = 0] = \frac{2}{3}$, which is always higher than the expected quality with $\beta = 1$. Therefore, even when the intermediary can charge consumers, it may prefer to be biased in favor of one firm and this results in a lower expected quality.

One implication of this result is that regulations that simply prohibit payment of commissions in return for recommendation may not be enough to rule out intermediary bias. Intermediaries can have an incentive to bias even when they collect revenues only from consumers.

7.2 Competing intermediaries

So far we have only considered structures with a single intermediary. The purpose of this section is to show that our main results are robust to the introduction of competition among intermediaries.

Suppose that there are two downstream firms, $N > 1$ intermediaries and, for simplicity, that all consumers are uninformed.[32] Each intermediary $n \in \{1, ..., N\}$ is consulted by μ_n consumers (consumers can consult at most one intermediary). The price that a firm has to pay to be recommended by intermediary n is p_n. Intermediaries can only recommend one firm. Consumers only observe the recommendation of the intermediary they consult. Let $\mu_{\mathcal{I}}$ be the mass of consumers who consult intermediaries who recommend firm $i \in \{1, 2\}$. A competitive equilibrium satisfies the following conditions:[33]

1. Firm i's quality choice r_i is optimal given $\mu_{\mathcal{I}}$.

2. The price p_n of intermediary n's recommendation is such that the recommended firm is better-off paying p_n and being recommended than not paying and not being recommended. The converse is true for the non-recommended firm.

One could either assume that consumers are naive, so that they choose an intermediary randomly and follow his recommendation (in which case the μ_n can be regarded as exogenous),

[32] The restriction to two downstream firms is inessential. Having only uninformed consumers allows us to focus on pure strategy equilibria.

[33] See Appendix A.5 for a detailed treatment of the competitive equilibrium.

or that they have rational expectations so that in equilibrium the only intermediaries with positive market shares are those who recommend the best firm. Below we discuss how either assumption affects our results.

First, it is instructive to look at the case in which both intermediaries' market shares and their recommendations are exogenous: $\mu_{\mathcal{I}}$ consumers use firm i and $\mu_{\mathcal{J}}$ use firm j.

Lemma 5. *Suppose that firm i expects to be used by $\mu_{\mathcal{I}}$ consumers. Then, (i) its optimal quality $\tilde{r}(\mu_{\mathcal{I}})$ is increasing in $\mu_{\mathcal{I}}$, and (ii) its profit $\tilde{\pi}(\mu_{\mathcal{I}})$ is increasing and convex.*

Proof. $\tilde{r}(\mu_{\mathcal{I}})$ is the solution to $K'(r) = \mu_{\mathcal{I}}$. By convexity of K, it is increasing in $\mu_{\mathcal{I}}$. Let $\pi(r, \mu) \equiv r\mu - K(r)$. We have $\tilde{\pi}(\mu_{\mathcal{I}}) = \pi(\tilde{r}(\mu_{\mathcal{I}}), \mu_{\mathcal{I}})$. By the envelope theorem, $\tilde{\pi}'(\mu_{\mathcal{I}}) = \tilde{r}(\mu_{\mathcal{I}}) > 0$ and $\tilde{\pi}''(\mu_{\mathcal{I}}) = \tilde{r}'(\mu_{\mathcal{I}}) > 0$. ■

Part (i) of the lemma extends the results of section 4, namely that a firm which benefits from a larger base of users through recommendations has more incentives to invest than its rival. If we relax the assumption that consumers are naive, but maintain exogenous recommendations by intermediaries,[34] we thus get the following result:

Proposition 6. *The game with exogenous recommendations and rational consumers has two kinds of equilibria, provided that each firm is recommended by at least one intermediary:*

1. *Asymmetric equilibria in which all the intermediaries with a positive market share recommend the same firm (say firm 1). Firm 1 chooses $r_1 = \hat{r} = \tilde{r}(1)$ and firm 2 chooses $r_2 = 0$.*

2. *Symmetric equilibria in which $\mu_{\mathcal{I}} = \mu_{\mathcal{J}} = 1/2$, and $r_1 = r_2 = \tilde{r}(1/2)$. These equilibria are unstable in the sense that best response dynamics following a small perturbation does not converge back to a symmetric equilibrium.*

Proof. Notice first that for each kind of equilibrium, there is a multiplicity of equilibria as the μ_n are not pinned down. It is straightforward to check that asymmetric and symmetric equilibria as described by 1 and 2 above are indeed equilibria. Let us now check that we cannot have an equilibrium with $0 < \mu_{\mathcal{J}} < \mu_{\mathcal{I}}$. If this was the case, then, by Lemma 5, firm i would choose a higher quality than firm j, and it would not be rational for consumers to ever follow an intermediary recommending j.

To see that a symmetric equilibrium is not stable, suppose that $\mu_{\mathcal{I}}$ increases to $1/2 + \epsilon$. Then, by Lemma 5, r_1 should be higher than r_2, which in turns should lead consumers away from firm 2. ■

[34] which means that we ignore part 2 of the definition of a competitive equilibrium.

Lemma 5 (i) and Proposition 6 can be useful to understand the forces at play in a situation where vertical structures (intermediaries and firms) compete with each other. For instance, in the search engine and maps market, broad strategic considerations probably prevent an outcome like integration of Bing with Google Maps, so that recommendations are better viewed as exogenous. The message here is that there is a scale advantage (Lemma 5) which, when coupled with rational expectations, leads to complete market tipping in equilibrium. The assumption of rational expectations, while standard in the literature, is perhaps too strong in the present context, as it implies that consumers are able to identify which intermediaries give the right recommendation through an unspecified mechanism. We therefore suggest caution in interpreting the market-tipping result of Proposition 6.

With endogenous recommendations, the equilibrium outcome no longer depends on the degree of rationality of consumers, and thus appears more robust. Indeed, we have the following result:

Proposition 7. *There exists a competitive equilibrium of the game with competing intermediaries and endogenous recommendations. In any equilibrium, all the intermediaries recommend the same firm (unanimity). This firm chooses the monopoly quality $\hat{r} = \tilde{r}(1)$ while its rival chooses $r = 0$, and both make zero profit. This results holds with naive consumers or with rational expectations.*

Proof. The complete proof is in Appendix A.5. The intuition for unanimity relies of part (ii) of Lemma 5. Suppose that consumers are naive so that the μ_n are exogenously given.[35] Suppose that $0 < \mu_{\mathcal{J}} < \mu_{\mathcal{I}}$. Firm i 's profit is given by $\tilde{\pi}(\mu_{\mathcal{I}})$, and firm j's is given by $\tilde{\pi}(\mu_{\mathcal{J}})$. Because $\tilde{\pi}$ is convex, we have $\hat{\pi} = \tilde{\pi}(1) > \tilde{\pi}(\mu_{\mathcal{I}}) + \tilde{\pi}(\mu_{\mathcal{J}})$, so that firm i would be strictly better-off had it bought the recommendations of the intermediaries who recommend j, contradicting the second equilibrium condition given above. ■

The competitive equilibrium described here is thus essentially the same as under a monopolistic intermediary.

7.3 Consumer search and product differentiation

Uninformed consumers in our baseline model are passive in the sense that they simply follow the intermediary's recommendation. We have seen that, conditional on the consumer being constrained to select a single firm, this behavior is rational. In some markets, though, one might expect uninformed consumers to search across alternatives—even after hearing

[35] With rational expectations unanimity is obtained by a logic similar to Proposition 6.

a recommendation. A simple way to study such situations is to introduce some horizontal differentiation, as well as a search cost for consumers, into the ex ante endorsement framework.

Suppose that the firms are located at the ends of a Hotelling segment of length 1. Consumers are distributed uniformly along the segment, and the disutility from consuming a product located at a distance x is tx. For analytical convenience, assume that the gross utility the consumer gets for product i is r_i. All consumers are initially uninformed about products' qualities,[36] and the intermediary advises them to visit firm 1 first. After observing r_1, consumers can decide to pay a search cost c and visit firm 2. If they do so, they observe r_2 and consume the product of the firm which delivers the highest net utility $r_i - tx_i$, where x_i is the distance to firm i.

Provided that t is large enough, firms play in pure strategies. Let r_i^* be the equilibrium quality of firm i. If a consumer located at x observes r_1 and anticipates firm 2 to play r_2^*, he will incur the search cost if and only if $r_1 - tx < r_2^* - t(1-x) - c$.[37] Therefore, firm 1 is used by a mass $\frac{t+c+r_1-r_2^*}{2t} \equiv x(r_1, r_2^*)$ of consumers if it chooses a quality r_1, while firm 2 is used by $1 - x(r_1, r_2^*)$ consumers if it chooses a quality larger than or equal to r_2^*. Firm 1's profit can therefore be written as

$$\pi_1 = r_1 x(r_1, r_2^*) - K(r_1),$$

while firm 2's profit, assuming $r_2 \geq r_2^*$, is

$$\pi_2 = r_2(1 - x(r_1, r_2^*)) - K(r_2).$$

The relevant first-order conditions for firms 1 and 2 are respectively

$$x_1(r_1^*, r_2^*) + r_1^* \frac{\partial x_1}{\partial r_1} = K'(r_1^*)$$

and

$$1 - x_1(r_1^*, r_2^*) = K'(r_2^*).$$

In this setup, we expect firm 1 to invest more than firm 2 for three reasons. First, notice that firm 2 cannot increase the mass of its users by choosing $r_2 > r_2^*$, because the only consumers who visit it are the ones who eventually use it. Firm 1, on the other hand, can dissuade consumers from searching by increasing r_1. Second, the previous effect is compounded by the fact that, as r_1 becomes larger than r_2, firm 1's market share increases, which leads it to invest even more so as to take advantage of the increased scale. Third, the existence of a search cost tilts the market share in favor of firm 1, which also increases its incentives to

[36] Adding informed consumers is straightforward but slightly more cumbersome.

[37] We break indifference by assuming that consumers do not search.

invest.

Indeed, with $K(r) = (r^2)/2$ and $t \geq \left(\sqrt{4c^2 + 4c + 9} + 2c + 3\right)/4$ (to guarantee an interior solution), we find

$$r_1^* = \frac{2t(t + c - 1)}{1 - 6t + 4t^2} > r_2^* = \frac{c - t(3 + 2c - 2t)}{1 - 6t + 4t^2}.$$

Even when search costs are zero, we find that firm 1 invests more than firm 2, as $r_1^* - r_2^*|_{c=0} = \frac{t}{1-6t+4t^2}$.

Compared to the objective benchmark, under which the intermediary sends consumers to the best firm, thereby eliminating the need to search, and in which in equilibrium firms choose $r_i = \frac{t}{2t-1}$, we find that ex ante endorsement leads firm 1 to invest more and firm 2 to invest less.

However, horizontal differentiation introduces an additional distortion: even if product 1 is of better quality than product 2, some consumers may still derive a greater utility form consuming the latter. While this may change the range of parameters under which ex ante or ex post contracts are better than the objective benchmark, the comparative statics with respect to investment incentives are unchanged compared to the baseline model.

8 Endorsement contracts and pricing by firms

8.1 Price and quality setting

We begin this section by considering a variation on the model above in which firms set both a quality, q, and a price p. Providing quality q has cost $K(q)$. Suppose that the $1 - \mu$ informed consumers know the value of the qs, but must visit a firm to learn its price. The uninformed consumers know neither, and follow the recommendation of the intermediary. Consumers can only visit one firm.[38] Each consumer has a downward-sloping demand $D(p, q)$, with choke price $\overline{p} = \min\{p : D(p, q) = 0\}$. Quality acts as a demand shifter: $D_2(p, q) > 0$.

The per-consumer revenue of a firm is $D(p, q)p$. Each firm sets the monopoly price, $p^m(q) \equiv \text{argmax}_p D(p, q)p$. Let $r(q) \equiv p^m(q)D(p^m(q), q)$. By a revealed preference argument, $r'(q)$ must be strictly positive. A consumer enjoys surplus

$$u(p, q) = \int_p^{\overline{p}} D(p, q)\, dp$$

from a product with price p and quality q.

[38]Allowing consumers to visit an extra firm at a cost would lead to the same result, per the *Diamond paradox* (Diamond, 1971).

The effect of an increase in q on consumer surplus is

$$\frac{d}{dq}u(p,q) = \int_{p^m}^{\overline{p}} D_2(p,q)\,dp - \frac{\partial p^m}{\partial q}D(p,q). \tag{11}$$

If (11) is positive then we have established that both u and r are increasing in quality so that the assumptions of the model described in section 2 are satisfied and the main results of sections 3–5 therefore remain unchanged. This is the case, for example, if $D(p,q) = q - p$.

The case in which (11) is negative, along with a variety of other interesting applications, is considered in the following subsection.

8.2 Harmful firm behavior

Let us now assume that $u'(r) < 0$ so that consumers' interests and those of firms are opposed. The preceding subsection showed how such a relationship can naturally arise in a model with pricing. Indeed, if the firms' products are homogeneous and exogenously fixed ($q_i = q_j = q$) with unit demad, then the model of Section 8.1 becomes one of pure pricing in which $r = p$ and $u'(r) < 0$ *always* holds. Since most attention in the literature on biased intermediaries has focused on this case of pure pricing,[39] it is interesting to consider how our results are influenced by reversing the sign of $u'(r)$. We show that, unlike in the "pure-investment" model considered above, the intermediary tends to recommend the wrong firm for consumers in both the ex ante and the ex post endorsement case when $u'(r) < 0$. Moreover, compared to an objective benchmark, the presence of a biased intermediary always increases r, which is harmful for consumers (corresponding, for example, to an increase in price).

To be more concrete, suppose that firms 1 and 2 produce a good that has some attribute whose intensity may be indexed by r. Each firm unilaterally determines the r for its product at cost $K(r)$, with $K'(r) \geq 0$. Without loss of generality, let r be the per-consumer revenue from a product with attribute r, and let $K(0) = 0$. Consumers' utility from a good with attribute r is $u(r)$, with $u'(r) < 0$. There are μ uninformed consumers who purchase the product recommended by an intermediary, while $1 - \mu$ informed consumers can observe attributes and buy the product with the lowest r.

Objective benchmark If the intermediary is committed to directing consumers towards the firm with the lowest r, the equilibrium is given by the Bertrand outcome: both firms choose $r = 0$. The intuition follows the conventional Bertrand logic: if $r_1 > 0$ then 2 can capture the whole market by slightly undercutting 1's choice of r; a process that continues

[39] See Inderst and Ottaviani (2012a), Armstrong and Zhou (2011) for other examples of such models.

until both firms earn zero profit.

Ex ante endorsement Let us now consider the following timing: At $t = 1$, firms bid to be endorsed by the intermediary. At $t = 2$, firms simultaneously choose their r, knowing which firm is endorsed. At $t = 3$, uninformed consumers go to the endorsed firm, informed consumers go to the firm with the lowest price.

The analysis of stages 2 and 3 amounts to an asymmetric version of Varian (1980), in which one firm has μ "loyal" consumers while the other has none; such an analysis is found in Narasimhan (1988) and need only be slightly modified for our purposes.

Narasimhan (1988) establishes that both firms must mix on the an interval $S = [\underline{r}, \overline{r}]$, where $\overline{r}$ solves $u(\overline{r}) = 0$. The only mass point is in F_1 at $\overline{r}$. The same is true in the present model. In particular, if there were no mass point at $\overline{r}$, or if the only mass point at $\overline{r}$ were in F_2 then $r_2 = \overline{r}$ would imply $r_2 > r_1$ and firm 2 would never attract any consumers; its profit would then be $-K(\overline{r})$.

Firm 1's profit from an arbitrary $r \in S$ is $\mu r + [1 - F_2(r)](1-\mu)r - K(r)$, which must equal that from setting $r_1 = \overline{r}$:

$$\mu r + [1 - F_2(r)](1-\mu)r - K(r) = \mu\overline{r} - K(\overline{r}).$$

Solving yields an expression for F_2:

$$F_2(r) = \frac{r - \mu\overline{r}}{r(1-\mu)} - \frac{K(r) - K(\overline{r})}{r(1-\mu)}.$$

Solving $F_2(r) = 0$ yields $\underline{r} = \mu\overline{r} - K(\overline{r}) + K(\underline{r})$, which implicitly pins down the value of $\underline{r}$. Lastly, we know that firm 2's profit when setting $\underline{r}$ must equal that from any other $r \in S$:

$$\begin{aligned}[1 - F_1(r)](1-\mu)r - K(r) &= (1-\mu)\underline{r} - K(\underline{r}) \\ &= (1-\mu)\left[\mu\overline{r} - K(\overline{r}) + K(\underline{r})\right] - K(\underline{r}),\end{aligned}$$

which implies

$$F_1(r) = \frac{r - \mu\overline{r}}{r} - \frac{K(r) - K(\overline{r})}{r(1-\mu)} - \frac{\mu\left[K(\overline{r}) - K(\underline{r})\right]}{r(1-\mu)}.$$

F_1 and F_2 define an equilibrium, provided that K does not increase too quickly over S so that the distributions are well-defined. Intuitively, K not increasing too quickly is necessary to ensure that firms' and consumers interests are really opposed—otherwise firms would prefer to set a low r to both attract consumers and economise on costs. It is straightforward to verify that $F_1(r) < F_2(r)$, so firm 1 chooses a (stochastically) higher r than does firm 2. Indeed, securing the endorsement allows firm 1 to face demand with a lower r-elasticity. Thus an important difference with the pure-investment model is that here the intermediary always endorses the “wrong” firm for consumers. Comparison with the objective case immediately reveals that endorsement causes both firms to strictly increase their choice of r.

Proposition 8. *Under ex ante endorsement with $u'(r) < 0$ the endorsed firm offers lower average utility to consumers than its rival. Endorsement unambiguously reduces consumer surplus.*

Ex post endorsement Suppose now that the timing is as follows: At $t = 1$, firms simultaneously choose r_i. At $t = 2$, the intermediary runs a second-price auction without reserve price. At $t = 3$, uninformed consumers go to the endorsed firm (the winner of the auction), and informed consumers go to the firm with the lowest r_i.

If a firm charges r_i, it wins the auction whenever $r_i > r_j$, which means that the intermediary always recommends the wrong firm.

Proposition 9. *Under ex post endorsement with $u'(r) < 0$ the endorsed firm offers lower utility to consumers than its rival. Endorsement unambiguously reduces consumer surplus.*

The expected profit of firm i, if it chooses a per-user revenue r_i and if firm j plays according to F, is

$$\int_0^{r_i} \mu(r_i - r_j)dF(r_j) + (1 - F(r_j))(1 - \mu)r_i - K(r_i)$$

The equilibrium is again symmetric in mixed strategies. Differentiating with respect to r_i over the support, we find that the equilibrium distribution is given by the following differential equation:

$$f(r) + \frac{1 - 2\mu}{(1 - \mu)r} F(r) = \frac{1 - \mu - K'(r)}{(1 - \mu)r} \tag{12}$$

along with a boundary condition ($F(\overline{r}) = 1$). For instance, in the case where firms only choose a price ($r_i = p_i$ and $K(r_i) = 0$), and where consumers have a unit demand with uniform willingness to pay $v = 1$, the solution to (12) is

$$F(r) = \frac{1 - \mu - \mu r^{\frac{2\mu - 1}{1 - \mu}}}{1 - 2\mu}$$

Discussion A natural application of the $u'(r) < 0$ model, as noted above, is to environments where intermediaries such as price comparison sites or other brokers help consumers to find low prices. As in Armstrong and Zhou (2011), we find that mis-selling is likely to be a problem in such environments and that endorsement will tend to inflate prices. What comparison of this example with the earlier ex ante quality-setting game shows, however, is that it is not quality- vs price-setting per se that is important in determining the effects of endorsement; rather, the crucial factor is the sign on the correlation between utility and revenue. Thus, the analysis in this section readily extends to dimensions other than prices that may harm consumers. For example, if firms can invest in developing privacy-invading technology that harms consumers but allows them to generate a database of consumer data that can be monetised then our model predicts that (i) relatively egregious invaders of privacy are more likely to be endorsed, and (ii) endorsement or vertical integration will tend to increase the overall level of privacy-invasion.

In general, the model with quality-setting ($u'(r) > 0$) suggests a less pessimistic outlook for endorsement than its counterpart with $u'(r) < 0$—especially in the ex ante case. Application of insights from the existing literature to competition issues such as merger control might, therefore, lead to an overly restrictive policy environment when quality is the main strategic variable. Our analysis suggests that regulators should be somewhat more circumspect about intervening in such instances.

9 Conclusion

Recent years have seen intermediaries in a broad range of contexts—from Internet search and retail to finance and medicine—subjected to scrutiny as regulators try to ascertain if and how contractual arrangements between these entities and downstream firms might affect the objectivity of advice and the incentives to invest. We investigate this issue in an environment where an intermediary is an essential gatekeeper for some consumers.

We find that when the primary role of the intermediary is to help consumers find the best product there is little danger that payments induce intermediaries to give bad advice. This is because the market mechanism by which firms compete to be endorsed tends to favour (or create) high-quality firms. This is in contrast to environments where the intermediary helps consumers find low prices, in which case there is an incentive to direct consumers to high-priced products where margins are larger.

Although we find limited cause for concern over the distortion of recommendations, the potential for endorsement contracts to affect investment incentives appears to be more pronounced. When endorsement is agreed prior to investment taking place, product quality can go up or down depending on whether there are economies of scale sufficient to offset the softened competition implied by endorsement. If, on the other hand, an endorsement contract is signed after firms invest then investment is unambiguously lower as firms anticipate that the intermediary will capture a share of the profits from quality provision.

A Omitted Proofs

A.1 Formal definitions

A strategy for firm i in this game is a CDF, $F_i \colon \mathbb{R}_+ \to [0,1]$ such that $F_i(r) = \Pr(r_i \le r)$. As usual, the support of F_i, $S(F_i)$, is the smallest closed set such that $\Pr[r_i \in S(F_i)] = 1$. It will be convenient to define $F_i^-(r) = \Pr(r_i < r)$.

A.2 Proof of Lemma 1

Claim 1. *In any equilibrium, there is no r such that both firms play r with positive probability.*

Proof. Suppose there exists an r played with positive probability by both firms. A tie then occurs with positive probability. We have $\pi_i(r) = (Pr[r_j < r] + Pr[r_j = r]/2)r - K(r)$. On the other hand, $\lim_{\epsilon \to 0^+} \pi(r+\epsilon) = (Pr[r_j < r] + Pr[r_j = r])r - K(r) > \pi_i(r)$, i.e. firm i has a profitable deviation. ■

Claim 2. *Both firms make zero profits in any equilibrium.*

Proof. Let r_{min} be the lowest quality in the support of either firm. Suppose without loss of generality that r_{min} is in firm 2's support. If it plays r_{min}, firm 2 does not serve any consumer and its profit is thus $-K(r_{min})$. So we must have $r_{min} = 0$, and firm 2 makes zero profit. Suppose now that firm 1 makes a positive profit, and let $\overline{r}_1$ be the largest quality in its support. For π_1 to be positive, we must have $\overline{r}_1(P[r_2 < \overline{r}_1] + \frac{P[r_2 = \overline{r}_1]}{2}) - K(\overline{r}_1) > 0$. Firm 2 can profitably deviate by playing a pure strategy $r_2 = \overline{r}_1 + \epsilon$ for ϵ small enough yet positive, because the previous inequality implies that $\overline{r}_1 - K(\overline{r}_1) > 0$ and thus $\overline{r}_1 + \epsilon - K(\overline{r}_1 + \epsilon) > 0$ by continuity of K. ■

Claim 3. *Both firms use mixed strategies in any equilibrium.*

Proof. Claim 1 establishes that there is no pure strategy equilibrium with $r_1 = r_2$. It suffices (by symmetry) to establish that (a) there is no pure strategy equilibrium with $r_2 < r_1$, and (b) that there is no equilibrium in which only firm 1 plays a pure strategy. To prove (a): $r_2 < r_1$ implies 1's profits are $r_1 - K(r_1)$, requiring $r_1 = \text{argmax}_x\{x - K(x)\}$. A deviation by 2 to $r_1 + \epsilon$, $\epsilon \to_+ 0$, thus yields profit $\lim_{\epsilon \to 0} r_1 + \epsilon - K(r_1 + \epsilon) = \max_x\{x - K(x)\} > 0$. This is profitable since two makes non-positive profit in the putative equilibrium.

To prove (b), suppose that $S(F_1) = r_1$ and that $F_2^-(r_1) > 0$. When setting an $r_2 < r_1$ 2's profits are $-K(r_2)$ so $S(F_2) \cap (0, r_1) = 0$. To rule out a profitable deviation for 2 it must be that $r - K(r) \le 0 \ \forall r > r_1$. This, though, implies $xr_1 - K(r_1) \le 0 \ \forall x \le 1$ and 1's profit, $r_1 P[r_2 = 0] - K(r_1)$, is non positive. That $S(F_2) \cap (0, r_1) = \emptyset$ along with concavity of 1's profits implies that 1 could strictly profit by deviating to $r_1 - \epsilon$, $\epsilon \to_+ 0$. This rules out $S(F_1) = r_1$ and $F_2^-(r_1) > 0$. It must, then, be the case that $F_2^-(r_1) = 0$, i.e. that $\underline{r}_2 \geq r_1$. Claim 1 implies $Pr[r_2 = r_1] = 0$ so 1 has the lowest quality for sure. It must then be the case that $r_1 = 0$ since 1's profits are $-K(r_1)$. 2's profits are $\overline{r}_2 - K(\overline{r}_2)$, requiring $\overline{r}_2 = \text{argmax}_x\{x - K(x)\}$. A deviation by 1 to $\overline{r}_2 + \epsilon$, $\epsilon \to_+ 0$, thus yields profit $\lim_{\epsilon \to 0} \overline{r}_2 + \epsilon - K(\overline{r}_2 + \epsilon) = \max_x\{x - K(x)\} > 0$ so 1 has a profitable deviation. ■

The next important feature of the equilibrium that will be established is that $S(F_i)$ is an interval. To do this, let us define r_i' as the smallest strictly positive $r_i \in S(F_i)$ such that $r_i' + \epsilon \notin S(F_i)$ for $\epsilon \to_+ 0$, and r_i'' as the smallest element of $S(F_i)$ such that $r_i'' > r_i'$ (when it exists). Thus (r_i', r_i'') is the (hypothetical) first gap in $S(F_i)$.

Claim 4. *(i) In equilibrium, if for some $r_i' < r_i''$, $S(F_i) \cap (r_i', r_i'') = \emptyset$, then $S(F_j) \cap (r_i', r_i'')$ has at most one element. (ii) Moreover, if $Pr[r_i = r_i''] = 0$, then $S(F_j) \cap (r_i', r_i'']$ has at most one element.*

Proof. Let $(r_j^1, r_j^2) \in (r_i', r_i'')^2$. We then have $F_i(r_j^1) = F_i(r_j^2)$. It follows that j's profit is $F_i(r_i')r_j - K(r_j)$ (which is concave in r_j) everywhere in (r_i', r_i'') and therefore that either (a) r_j^1 and r_j^2 do not yield the same profit or (b) there is an $r \in (r_j^1, r_j^2)$ that yields higher profit than either r_j^1 or r_j^2. In either (a) or (b) it is clear that we can't have $(r_j^1, r_j^2) \in S(F_j)^2$. Applying this result recursively to eliminate elements of $S(F_j)$ implies (i). The proof of part (ii) is identical with $(r_j^1, r_j^2) \in (r_i', r_i'']^2 \implies F_i(r_j^1) = F_i(r_j^2)$. ■

Claim 5. *There is no equilibrium with* $S(F_i) \cap (r_i', r_i'') = \emptyset$ and $Pr[r_j = r_i''] = 0$.

Proof. Suppose that there is an equilibrium with $S(F_i) \cap (r_i', r_i'') = \emptyset$ and $Pr[r_j = r_i''] = 0$. By Claim 4, there exists an $\epsilon > 0$ such that $\Pr(r_i'' - \delta > r_j) = F_j(r_i'')$ for all $\delta \leq \epsilon$. It follows that the left-hand derivative of π_i at r_i'' is of the form $F_j(r_i'') - K'(r_i'') \geq 0$; to prevent i from deviating to some $r_i'' - \epsilon$ it must be the case that this is positive. Firm i's profit when setting r_i'' is $F_j(r_i'')r_i'' - K(r_i'') = 0$ (where the zero profit condition follows from Claim 2). Combining this zero profit condition with $F_j(r_i'') - K'(r_i'') \geq 0$ yields $K(r_i'') \geq K'(r_i'')r_i''$, which can never hold since K is convex. We thus have a contradiction. ■

Claim 6. *There is no equilibrium with* $S(F_i) \cap (r_i', r_i'') = \emptyset$ and $Pr[r_j = r_i''] > 0$.

Proof. Suppose that there is an equilibrium with $S(F_i) \cap (r_i', r_i'') = \emptyset$ and $Pr[r_j = r_i''] > 0$. We know from Claim 1 that $Pr[r_i = r_i''] = 0$. Thus, by Claim 4 $S(F_j) \cap (r_i', r_i'') = \emptyset$. We know that $r_i' \in S(F_j)$ because if this were not the case there would be a gap in $S(F_j)$ at $(r_i' - \epsilon, r_i')$; Claim 4 would then imply that there is also a gap in $S(F_i)$ below r_i', contradicting the assumption that (r_i', r_i'') is the first gap. That $r_i' \in S(F_j)$ implies $Pr[r_i = r_i'] = 0$—otherwise j

would prefer to deviate to $r_i' + \epsilon$ for $\epsilon \to_+ 0$. So far it has been established that an equilibrium with $S(F_i) \cap (r_i', r_i'') = \emptyset$ *and* $Pr[r_j = r_i''] > 0$ must be such that there is an equal sized gap in each of F_i and F_j at (r_i', r_i''), and that firm i puts no mass on either endpoint of this interval. For any $r_j \in [r_i', r_i'']$, firm j's profits are therefore $F_i(r_i')(r_j) - K(r_j)$, which (by concavity) can be maximised at only one point. Since j puts positive mass on r_i'', this maximum must occur at r_i'': $\pi_j(r_i') < \pi_j(r_i'')$. Thus, r_i' cannot be in the support of j—a contradiction. ■

Given that $r_i' > 0$ by assumption, Claims 5 and 6 do not rule out the case in which $S(F_i) = \{0\} \cup [\underline{r}, \overline{r}]$. This is achieved in the following Claim:

Claim 7. $S(F_i) = [\underline{r}_i, \overline{r}_i]$.

Proof. Recall that the definition of r_i' held that it be strictly positive. Claims 5 and 6 therefore rule out all gaps except those of the form $(0, r_i'')$. Notice that having $S(F_i) = \{0\} \cup [\underline{r}, \overline{r}]$ and $P[r_i = 0] = 0$ is essentially the same as having $S(F_i) = [\underline{r}, \overline{r}]$, so we will prove that $S(F_i) = \{0\} \cup [\underline{r}, \overline{r}]$ and $P[r_i = 0] > 0$ lead to a contradiction. Suppose then that we have $S(F_i) = \{0\} \cup [\underline{r}, \overline{r}]$ and $P[r_i = 0] > 0$. Claim 1 thus implies $0 \notin S(F_j)$. We also know that $\underline{r}_j = r_i''$: $\underline{r}_j > r_i''$ would result in negative profits for i when setting r_i''; $\underline{r}_j < r_i''$ would, by Claim 4, imply a gap in $S(F_j)$ at $(\underline{r}_j, r_i'')$ (which is impossible by Claims 5 and 6).

It must be that $Pr[r_i = r_i''] = 0$. Indeed, suppose $Pr[r_i = r_i''] > 0$. By Claim 1 and since $\underline{r}_j = r_i''$, we would then have $F_j(r_i'') = 0$, which would then imply $\pi_i(r_i'') = -K(r_i'') < 0$, a contradiction.

To complete the proof, note that $r_i'' \in S(F_j) \implies \pi_j(r_i'') = F_i(r_i'')r_i'' - K(r_i'') = 0$ (by Claim 2). Moreover, $Pr[r_i = r_i''] = 0$ implies that the left hand derivative of π_j evaluated at r_i'' must be positive—otherwise j would wish to deviate to some $r < r_i''$. Thus, $F_i(r_i'') - K'(r_i'') \geq 0$. Combining this derivative with the expression for profits at r_i'' yields $K(r_i'')/r_i'' \geq K'(r_i'')$, which is never true since K is convex). ■

Claim 8. $S(F_1) = S(F_2) = [0, \overline{r}]$.

Proof. First, establish that $\underline{r}_1 = \underline{r}_2 = 0$. If $\underline{r}_1 \geq \underline{r}_2$ then 2, when setting $r_2 = \underline{r}_2$, earns profit $-K(\underline{r}_2)$. This, in turn, implies $\underline{r}_2 = 0$ and $(0, \underline{r}_1) \cap S(F_1) = \emptyset$. Such a possibility is ruled out by Claim 7. If $\underline{r}_1 = \underline{r}_2 > 0$ then, by Claim 1, at least one i has $F_j(\underline{r}_i) = 0 \implies \pi_i = -K(\underline{r}_i)$ and would prefer to deviate to a lower r.

Secondly, show that $\overline{r}_1 = \overline{r}_2 = \overline{r}$. Suppose $\overline{r}_1 > \overline{r}_2$. We would then have $\pi_1(r) = r - K(r)\ \forall r \in (\overline{r}_2, \overline{r}_1]$. Since this is concave, there can be at most one r_1 in this interval that maximises 1's profits. There are two possibilities: (a) $\widehat{r} \leq \overline{r}_2$ would imply 1's profits are decreasing for $r_1 \in (\overline{r}_2, \overline{r}_1]$ so 1 would prefer to reduce $\overline{r}_1$; (b) $\widehat{r} > \overline{r}_2$ would require $\overline{r}_1 = \widehat{r}$, but then 2 could deviate to $\widehat{r} + \epsilon$ and get the monopolist profit (when ϵ is small). ■

A.3 Proof of Lemma 3

Remark 1. *The proof of Claim 1 applies verbatim to the ex ante contracting game.*

Claim 9. *There is no equilibrium in pure strategies.*

Proof. Pure strategy equilibria with $r_1 = r_2$ are ruled out by Claim 1. Suppose $r_1 > r_2 \implies \pi_2 = -K(r_) \implies r_2 = 0$. It follows that $\pi_1 = r_1 - K(r_1)\ \forall r_1 > 0$ so for r_1 to be optimal $r_1 = \widehat{r}$ must hold. However, 2 could then profitably deviate to $\widehat{r} + \epsilon$ (ϵ small) and earn $\pi_2 = (1-\mu)(\widehat{r} + \epsilon) - K(\widehat{r} + \epsilon)$. This is positive when $\mu < 1 - K(\widehat{r})/\widehat{r}$. The last possibility is $r_2 > r_1 \implies \pi_2 = (1-\mu)r_2 - K(r_2)$. For 2 not to want to deviate it must be true that $(1-\mu) = K'(r_2) \implies \mu < \widehat{r}$. But then 1 could deviate to $\widehat{r}$ and capture all consumers, which must increase its profits since $\widehat{r}$ is the globally profit-maximising quality. ∎

Claim 10. $\min S(F_1) \equiv \underline{r}_1 \neq 0$

Proof. $0 \in S(F_1) \implies \pi_1 = 0$. This cannot be part of an equilibrium since 1 could always make positive profit by catering only to its μ captive consumers and maximizing $\mu r - K(r)$. ∎

Claim 11. *(1) $S(F_2) \cap (0, \underline{r}_1) = \emptyset$ and (2) $Pr[r_2 = \underline{r}_1] = 0$.*

Proof. (1) $r_2 \in (0, \underline{r}_1) \implies F_1(r_2) = 0 \implies \pi_2 = -K(r_2)$ (by Claim 10). Such a strategy is therefore dominated by $r_2 = 0$. (2) A mass point in both F_1 and F_2 is ruled out by Claim 1. Thus, if $Pr[r_2 = \underline{r}_1] > 0$, $F_1(\underline{r}_1) = 0 \implies \pi_2 = -K(\underline{r}_1) < 0$. ∎

Claim 12. *$\underline{r}_1$ solves $K'(\underline{r}_1) = \mu + (1-\mu)F_2(0)$.*

Proof. Claim 11 implies $F_2(\underline{r}_1) = F_2(0)$ and that firm 1's profits are $[\mu + (1-\mu)F_2(0)]\, r - K(r)$ for every $r \in (0, \underline{r}_1)$. If $K'(\underline{r}_1) > \mu + (1-\mu)F_2(0)$ then it follows that 1 could profitably deviate by setting some $r \in (0, \underline{r}_1)$. The right-hand derivative of 1's profits at $\underline{r}_1$ is

$$\mu + (1-\mu)F_2(0) - K'(\underline{r}_1) + (1-\mu)\underline{r}_1 F_2'(\underline{r}_1) \geq \mu + (1-\mu)F_2(0) - K'(\underline{r}_1).$$

Thus, if we had $K'(\underline{r}_1) < \mu + (1-\mu)F_2(0)$, firm 1's profit would be increasing in r around $\underline{r}_1$, and thus $\underline{r}_1$ would not be in $S(F_1)$. ∎

Claim 13. $\max S(F_1) \equiv \overline{r}_1 = \max S(F_2) \equiv \overline{r}_2 \equiv \overline{r} > \widehat{r}$. *$Pr[r_i = \overline{r}] = 0 \forall i$.*

Proof. Suppose first that $\overline{r}_1 > \overline{r}_2$. For any $r_1 > \overline{r}_2$, firm 1's profit is $r_1 - K(r_1)$. Therefore, by concavity, $S(F_1) \cap (\overline{r}_2, \overline{r}_1] = \{\overline{r}_1\}$. Indeed, no two points can generate the same level of profit unless they are dominated by a third one. There are two possibilities to consider:

1. $\overline{r}_2 \geq \widehat{r}$. In that case, because $r \mapsto r - K(r)$ is decreasing over $(\widehat{r}, +\infty)$, firm 1 would be better-off choosing a quality $r \in (\overline{r}_2, \overline{r}_1)$.

2. $\overline{r}_2 < \widehat{r}$. Firm 1's best response is to choose $\overline{r}_1 = \widehat{r}$. But then $\pi_1(\overline{r}_1) > \pi_1(r)$ for any $r \neq \overline{r}_1$, i.e. firm 1 must play a pure strategy, which is ruled out by Claim 9.

We have thus proven that $\overline{r}_1 \leq \overline{r}_2$. The proof that $\overline{r}_1 \geq \overline{r}_2$ is the mirror image of the previous one, replacing $\widehat{r}$ by the r that solves $K'(r) = 1 - \mu$. We thus have $\overline{r}_1 = \overline{r}_2 = \overline{r}$.

Suppose now that firm i has a mass point at $\overline{r}$. Then, firm j could cause an upward jump in its profit by putting some mass at $\overline{r} + \epsilon$, $\epsilon \to_+ 0$. Finally, $\overline{r} > \widehat{r}$ must hold, otherwise firm 1 could deviate and choose $r_1 = \widehat{r}$ with probability 1. ■

Claim 14. *Firm 2 plays* 0 *with positive probability, and thus makes zero profit in equilibrium.*

Proof. Suppose $F_2(0) = 0$. Claim 11 implies that $F_2(\underline{r}_1) = 0$. Since firm 1 is indifferent over its support, we must have $\pi_1(\underline{r}_1) = \pi_1(\overline{r})$, i.e. $\mu \underline{r}_1 - K(\underline{r}_1) = \overline{r} - K(\overline{r})$. Put differently,

$$K(\overline{r}) - K(\underline{r}_1) = \overline{r} - \mu \underline{r}_1. \tag{13}$$

Given that $\overline{r} \in S(F_2)$ and that F_1 has no mass at $\overline{r}$ (see claim 13), we must have $\pi_2(\overline{r}) = (1-\mu)\overline{r} - K(\overline{r}) \geq \lim_{\epsilon \to 0^+} \pi_2(\underline{r}_1 + \epsilon) = (1-\mu)Pr[r_1 = \underline{r}_1]\underline{r}_1 - K(\underline{r}_1)$, i.e.

$$K(\overline{r}) - K(\underline{r}_1) \leq (1-\mu)(\overline{r} - Pr[r_1 = \underline{r}_1]\underline{r}_1). \tag{14}$$

Combining (13) and (14) leads to $\mu\overline{r} \leq \mu\underline{r}_1 - (1-\mu)Pr[r_1 = \underline{r}_1]\underline{r}_1$, a contradiction. ■

Claim 15. *The common upper-bound* $\overline{r}$ *solves* $(1-\mu)\overline{r} = K(\overline{r})$.

Proof. If firm 2 sets $r = \overline{r}$, given that $Pr[r_1 = \overline{r}] = 0$, firm 2's expected profit is $(1-\mu)\overline{r} - K(\overline{r})$. By Claim 14, this must be zero. ■

Claim 16. *There exists a unique value of* $F_2(0)$ *consistent with equilibrium behavior.*

Proof. By firm 1's indifference condition, we have

$$[\mu + (1-\mu)F_2(0)]\,\underline{r}_1 - K(r_1) = \overline{r} - K(\overline{r}) \tag{15}$$

Using Claim 12 (which implies a bijective relationship between $\underline{r}_1$ and $F_2(0)$), the derrivative of the left-hand side of (15) with respect to $F_2(0)$ is

$$(1-\mu)\underline{r}_1 + \underbrace{[\mu + (1-\mu)F_2(0) - K'(\underline{r}_1)]}_{=0,\text{ by claim 12}} \frac{d\underline{r}_1}{dF_2(0)} > 0.$$

The left-hand side of (15) is thus increasing in $F_2(0)$. Given that $\overline{r}$ is uniquely defined (claim 15), the right-hand side of (15) does not vary with $F_2(0)$. There is thus a unique value of $F_2(0)$ that solves (15). ■

Remark 2. *The proof of Claim 4 applies verbatim to the ex ante contracting game.*

Claim 17. *The lowest positive quality in the support of F_2 satisfies* $\min\{S(F_2)\setminus\{0\}\} \equiv \underline{r}_2 = \underline{r}_1 \equiv \underline{r}$.

Proof. $\underline{r}_2 < \underline{r}_1$ is ruled out by Claim 11. Suppose that $\underline{r}_2 > \underline{r}_1$. Claim 4, implies $S(F_1) \cap (0, \underline{r}_2)$ has at most one element. By definition, $\underline{r}_1 \in S(F_1)$, so $S(F_1) \cap (\underline{r}_1, \underline{r}_2) = \emptyset$. The fact that no point in $(\underline{r}_1, \underline{r}_2)$ is played by either firm implies that $r_2(1-\mu)Pr[r_1 = r_2] - K(r_2)$ is non-decreasing at $\underline{r}_2$ (otherwise firm 2 would deviate to a lower $\underline{r}_2$). Taking the derivative, we thus have $(1-\mu)Pr[r_1 = \underline{r}_2] \geq K'(\underline{r}_2)$. Now, if $\underline{r}_2 \in S(F_2)$, we must have, by claim 14, $(1-\mu)Pr[r_1 = \underline{r}_2]\underline{r}_2 - K(\underline{r}_2) = 0$. Combining the two previous equations, we obtain $K(\underline{r}_2)/\underline{r}_2 \geq K'(\underline{r}_2)$, which is impossible by convexity of K. ■

Claim 18. $[\underline{r}, \overline{r}] \subseteq S(F_i)\,,\ i = 1, 2.$

Proof. Let $i = 2$, $j = 1$. The proofs of Claims 5 and 6 then apply verbatim to the ex ante contracting game.This implies $[\underline{r}, \overline{r}] \subseteq S(F_2)$. To show $[\underline{r}, \overline{r}] \subseteq S(F_1)$ suppose that $(r_1', r_1'') \subseteq [\underline{r}, \overline{r}]$, $(r_1', r_1'') \nsubseteq S(F_1)$. Claim 4 implies that $S(F_2)$ has at most one point in (r_1', r_1''), which implies $(r_1', r_1'') \nsubseteq S(F_2)$. This contradicts Claims 5 and 6. ■

Claim 19. *Neither F_1 nor F_2 have a mass point on $(\underline{r}, \overline{r}]$.*

Proof. By claim 13, we already know that $\overline{r}$ cannot be a mass point. If $r \in (\underline{r}, \overline{r})$ was played with positive probability by firm i, firm j could increase its profit by having a mass point at $r + \epsilon$, $\epsilon \to_+ 0$. ■

A.4 Proof of Proposition 2

Comparison of (1) and (2) with Lemma 1 reveals

Corollary 2. *The benchmark case yields an identical quality distribution to the ex ante contracting case when $\mu \to 0$.*

Thus, comparing ex ante contracting to the benchmark amounts to a comparative static analysis with respect to μ under ex ante contracting. The following claims establish the result:

Proposition 10. $F_2(r) - F_1(r)$ *is non-negative and increasing in μ.*

Proof. For all $r \in [\underline{r}, \overline{r})$, using (2), Claim 15, and (1),

$$F_2(r) = \frac{K(r)}{(1-\mu)r} - \frac{\mu r - \overline{r} + K(\overline{r})}{(1-\mu)r} = F_1(r) - \frac{\mu(r-\overline{r})}{(1-\mu)r}$$

for every $r \in S_1 S(F_1)$. It follows that

$$F_2(r) - F_1(r) = \frac{\mu(\overline{r}-r)}{(1-\mu)r} > 0 \text{ and } \frac{\partial F_2(r) - F_1(r)}{\partial \mu} > 0.$$

The proof is completed by noting $F_1(r) = 0 \leq F_2(r)$ for $r < \underline{r}$ and $\partial F_2(0)/\partial\mu > 0$. ■

Claim 20. $\partial F_2(r)/\partial\mu > 0 \forall r < \overline{r}$

Proof. $F_2(0)$ is given in (5). In general, a function $F(x)/G(x)$ is increasing in x if and only if $F'(x)/F(x) > G'(x)/G(x)$. $F_2(0)$ is therefore increasing in μ if $[K''(\underline{r}) - 1] / [K'(\underline{r}) - \mu] > -1/(1-\mu)$. We know the numerator on the left hand side is larger than that on the right since $K''(r) > 0$. We know the denominator on the right is larger than that on the left since $\underline{r} < \widehat{r} \implies K'(\underline{r}) < K'(\widehat{r}) = 1$.

It thus remains to show that $\partial F_2(r)/\partial\mu > 0 \forall r \in (0, \overline{r})$. $F_2(r)$ is given by (2). The denominator is decreasing in μ so it suffices to show that the numerator is increasing. Differentiating the numerator yields

$$\overline{r} - r + \mu\frac{\partial\overline{r}}{\partial\mu}. \tag{16}$$

Totally differentiating the expression for $\overline{r}$ from Claim 15 yields

$$d\overline{r}(1-\mu) - \overline{r}d\mu = K'(\overline{r})d\overline{r} \implies \frac{\partial\overline{r}}{\partial\mu} = \frac{\overline{r}}{(1-\mu) - K'(\overline{r})}.$$

Substituting this into (16) yields

$$\overline{r}\left[1 + \frac{\mu}{(1-\mu) - K'(\overline{r})}\right] - r = \overline{r}\underbrace{\frac{1-K'(\overline{r})}{(1-\mu) - K'(\overline{r})}}_{>1} - r.$$

Since $\overline{r} \geq r$ for all $r \in (\underline{r}, \overline{r}]$ this is positive. ■

Claim 21. $\partial\underline{r}/\partial\mu > 0$, $\partial\overline{r}/\partial\mu > 0$.

Proof. From Claim 12 we know that $\underline{r}$ is increasing in μ if and only if $\mu + (1-\mu)F_2(0)$ is increasing in μ. That this is the case can easily be verified given Claim 20. From Claim 15 we have $1-\mu = K(\overline{r})/\overline{r}$ so that $\overline{r}$ is decreasing in μ if and only if $K(\overline{r})/\overline{r}$ is increasing in $\overline{r}$. Differentiating $K(\overline{r})/\overline{r}$ yields $[\overline{r}K'(\overline{r}) - K(\overline{r})]/r^2$, which is positive since K is convex. ■

Claim 22. *An increase in μ causes a rotation in F_1. Formally, suppose μ is increased to $\mu' > \mu$ with corresponding $S(F_1) = [\underline{r}', \overline{r}']$. The result is that $F_1(r)$ decreases for any $r < \underline{r}'$ and increases for any $r \geq \underline{r}'$.*

Proof. By definition, we have $F_1(r,\mu) \geq F_1(r,\mu') = 0 \forall r < \underline{r}'$, (with strict inequality for $r \in (\underline{r}, \underline{r}')$), and $F_1(r,\mu) = \min\{K(r)/(1-\mu)r, 1\} \leq \min\{K(r)/(1-\mu')r, 1\} = F_1(r,\mu') \, \forall r \geq \underline{r}'$, (with strict inequality for $r \in (\underline{r}', \overline{r})$. ■

A.5 Proof of Proposition 7

Let $\mathbf{x}_i \equiv (x_{i,1}, \ldots, x_{i,N})$, with $x_{i,n} = 1$ if firm i is endorsed by intermediary n, and 0 otherwise. Let $\mathbf{p} \equiv (p_1, \ldots, p_N)$ be the vector of prices for each intermediary's endorsement.

A competitive equilibrium with rational expectations is a tuple $(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}, r_1, r_2, \boldsymbol{\mu})$ such that:

1. The allocation of endorsement $\mathbf{x}_i$ and of attention $\boldsymbol{\mu}$ are feasible:

$$\forall n \in \mathcal{N}, (x_{1,n}, x_{2,n}) \in \{0,1\}^2 \text{ and } x_{1,n} + x_{2,n} = 1; \quad \mu_n \geq 0 \text{ and } \sum_{n \in \mathcal{N}} \mu_n = 1.$$

2. Firm i's quality, r_i, maximizes i's profit given the set of intermediaries who endorse it: $r_i = \tilde{r}(\mathbf{x}_i \cdot \boldsymbol{\mu})$.

3. Firm i's demand for endorsement maximizes its profit:

$$\tilde{\pi}(\mathbf{x}_i \cdot \boldsymbol{\mu}) - \mathbf{x}_i \cdot \mathbf{p} \geq \tilde{\pi}(\mathbf{y}_i \cdot \boldsymbol{\mu}) - \mathbf{y}_i \cdot \mathbf{p} \text{ for all } \mathbf{y}_i \in \{0,1\}^N.$$

4. Consumers choose their intermediary rationally: $\mu_n > 0$ and $x_{i,n} = 1 \implies r_i \geq r_j$.

A competitive equilibrium with naive consumers is a tuple $(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}, r_1, r_2)$ such that conditions 1-3 above hold, while $\boldsymbol{\mu}$ is exogenously given. The proof below does not use of condition 4, so that the result holds with both naive and rational consumers.

We say that an allocation of endorsement is *unanimous* when intermediaries 1 to N endorse the same firm.

Let us first show that any equilibrium involves unanimous endorsement. Suppose that this is not the case, and that a non-empty subset of intermediaries $\mathcal{I}_1$ endorse firm 1 while a subset $\mathcal{I}_2$ endorse firm 2. For this to be an equilibrium, 1 must prefer to be endorsed by $\mathcal{I}_1$ rather than by $\mathcal{N}$ given the prices of endorsement. This implies

$$\tilde{\pi}(\mu_{\mathcal{I}_1}) - \sum_{n \in \mathcal{I}_1} p_n \geq \tilde{\pi}(1) - \sum_{n \in \mathcal{N}} p_n.$$

Firm 2 must prefer to be endorsed by $\mathcal{I}_2$ rather than by no one, so that

$$\tilde{\pi}(\mu_{\mathcal{I}_2}) - \sum_{n \in \mathcal{I}_2} p_n \geq 0.$$

Summing these inequalities leads to $\tilde{\pi}(\mu_{\mathcal{I}_1}) + \tilde{\pi}(\mu_{\mathcal{I}_2}) \geq \tilde{\pi}(1)$, a contradiction because $\tilde{\pi}$ is convex.

Now suppose that a unanimous equilibrium exists. By definition of an equilibrium (condition 2), the endorsed firm (say firm 1) must choose a quality $\tilde{r}(1)$, which is the monopoly quality. In a unanimous equilibrium, firm 1 must be better-off than if it was endorsed by nobody:

$$\tilde{\pi}(1) - \sum_{n \in \mathcal{N}} p_n \geq 0.$$

Firm 2 must be better-off than if it was endorsed by everybody:

$$0 \geq \tilde{\pi}(1) - \sum_{n \in \mathcal{N}} p_n,$$

which proves that both firms make zero profit.

For any attention vector $\boldsymbol{\mu}$, the following vector of prices supports an equilibrium:

$$p_1 = \tilde{\pi}(\mu_1), \text{ and } \forall n > 1, p_n = \tilde{\pi}(\mu_1 + ... + \mu_n) - \tilde{\pi}(\mu_1 + ... + \mu_{n-1})$$

Indeed, one can first check that both firms make zero profit if 1 is endorsed by $\mathcal{N}$ and 2 by nobody. To show that we have an equilibrium, let us check that being endorsed by a subset $\mathcal{I}$ of the intermediaries cannot increase a firm's profit. Let $(a_1, ..., a_k)$ be the indexes of the k intermediaries in $\mathcal{I}$. We claim that

$$\tilde{\pi}(\mu_{a_1} + ... + \mu_{a_k}) - (p_{a_1} + ... + p_{a_k}) \leq \tilde{\pi}(\mu_{a_1} + ... + \mu_{a_{k-1}}) - (p_{a_1} + ... + p_{a_{k-1}})$$

Indeed the previous inequality simplifies to

$$\tilde{\pi}(\mu_{a_1} + ... + \mu_{a_k}) - \tilde{\pi}(\mu_{a_1} + ... + \mu_{a_{k-1}}) \leq p_{a_k}$$

By definition, $p_{a_k} = \tilde{\pi}(\mu_1 + ... + \mu_{a_k}) - \tilde{\pi}(\mu_1 + ... + \mu_{a_k-1})$. By convexity of $\tilde{\pi}$, this must be larger than $\tilde{\pi}(\mu_{a_1} + ... + \mu_{a_k}) - \tilde{\pi}(\mu_{a_1} + ... + \mu_{a_{k-1}})$, which proves our claim.

Repeating the argument, we get $\tilde{\pi}(\mu_{a_1} + ... + \mu_{a_k}) - (p_{a_1} + ... + p_{a_k}) \leq \tilde{\pi}(\mu_{a_1}) - p_{a_1} \leq 0$, the last inequality also following from the convexity of $\tilde{\pi}$. Thus, no firm can do better than zero profit.

Finally, it is clear that, under unanimous endorsement, consumers are indifferent with respect to which intermediary to listen to.

References

All Things D (2011). *Google Will Pay Mozilla Almost $300M Per Year in Search Deal, Besting Microsoft and Yahoo.* URL: `http://allthingsd.com/20111222/google-will-pay-mozilla-almost-300m-per-year-in-search-deal-besting-microsoft-and-yahoo/`.

Armstrong, Mark, John Vickers, and Jidong Zhou (2009). "Prominence and Consumer Search". *RAND Journal of Economics* 40.2, pp. 209–233.

Armstrong, Mark and Jidong Zhou (2011). "Paying for Prominence". *Economic Journal* 121.556, F368–F395.

Athey, Susan and Glenn Ellison (2011). "Position Auctions with Consumer Search". *Quarterly Journal of Economics* 126.3, pp. 1213–1270.

Baye, Michael R, Dan Kovenock, and Casper G De Vries (1996). "The all-pay auction with complete information". *Economic Theory* 8.2, pp. 291–305.

Berndt, Ernst R. (2002). "Pharmaceuticals in U.S. Health Care: Determinants of Quantity and Price". *Journal of Economic Perspectives* 16.4, pp. 45–66.

Biglaiser, Gary (1993). "Middlemen as Experts". *RAND Journal of Economics* 24.2, pp. 212–223.

Biglaiser, Gary and James W Friedman (1994). "Middlemen as guarantors of quality". *International Journal of Industrial Organization* 12.4, pp. 509–531.

Bloomberg (2014). *Google Sued Over Claims Android Tie-Ins Hurt Competition.* URL: `http://www.bloomberg.com/news/2014-05-01/google-sued-over-claims-android-device-accords-hurt-competition.html`.

Bolton, Patrick and Michael D. Whinston (1993). "Incomplete Contracts, Vertical Integration, and Supply Assurance". *Review of Economic Studies* 60.1, pp. 121–148.

Buehler, Benno and Florian Schuett (forthcoming). "Certification and minimum quality standards when some consumers are uninformed". *European Economic Review.*

Caves, Richard E (2000). *Creative industries: Contracts between art and commerce.* 20. Harvard University Press.

Chen, Yongmin and Chuan He (2011). "Paid Placement: Advertising and Search on the Internet". *Economic Journal* 121.556, F309–F328.

Choi, Jay Pil (2004). "Tying and Innovation: A Dynamic Analysis of Tying Arrangements". *Economic Journal* 114, pp. 83–101.

Choi, Jay Pil and Byung-Cheol Kim (2010). "Net Neutrality and Investment Incentives". *RAND Journal of Economics* 41.3, pp. 446–471.

Cremer, Helmuth and Philippe De Donder (2013). "Network Investment under Legal and Ownership Unbundling". *Review of Network Economics* 12.1, pp. 27–59.

De Cornière, Alexandre and Greg Taylor (forthcoming). "Integration and Search Engine Bias". *RAND Journal of Economics*.

Dellarocas, Chrysanthos (2005). "Reputation mechanism design in online trading environments with pure moral hazard". *Information Systems Research* 16.2, pp. 209–230.

Diamond, Peter (1971). "A Model of Price Adjustment". *Journal of Economic Theory* 3.2, pp. 156–168.

Drugov, Mikhail and Dmitry Ryvkinz (2013). "Contests with arbitrary favorites". Working Paper.

Durbin, Erik and Ganesh Iyer (2009). "Corruptible advice". *American Economic Journal: Microeconomics* 1.2, pp. 220–242.

Economides, Nicholas and Benjamin E Hermalin (2012). "The economics of network neutrality". *The RAND Journal of Economics* 43.4, pp. 602–629.

Edelman, Benjamin G (2014). "Leveraging Market Power Through Tying and Bundling: Does Google Behave Anti-Competitively?" *Harvard Business School NOM Unit Working Paper* 14-112.

Edelman, Benjamin and Zhenyu Lai (2013). "Exclusive Preferential Placement as Search Diversion: Evidence from Flight Search". Working Paper.

Engelberg, Joseph, Christopher A. Parsons, and Nathan Tefft (2013). "First, Do No Harm: Financial Conflicts in Medicine". Working Paper.

European Comission (2014). *Antitrust: Commission obtains from Google comparable display of specialised search rivals.* URL: `http://europa.eu/rapid/press-release_IP-14-116_en.htm`.

Fair Search (2014). *Fairsearch Europe Urges European Commission to Make Google Proposal Public, Proposal Reinforces Google's Dominant Position Instead Of Remedying Problems.* URL: `http://www.fairsearch.org/search-manipulation/fairsearch-europe-urges-european-commission-to-make-google-proposal-public-proposal-reinforces-googles-dominant-position-instead-of-remedying-problems/`.

Federico, Giulio and Pierre Régibeau (2011). “Abusive incentive contracts: British Airways revisited”. Working Paper.

Franke, Jörg et al. (2013). “Effort maximization in asymmetric contest games with heterogeneous contestants”. *Economic Theory* 52.2, pp. 589–630.

Grossman, Sanford J and Oliver D Hart (1986). “The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration”. *Journal of Political Economy* 94.4, pp. 691–719.

Hagiu, Andrei and Bruno Jullien (2011). “Why Do Intermediaries Divert Search?” *RAND Journal of Economics* 42.2, pp. 337–362.

Inderst, Roman and Marco Ottaviani (2012a). “Competition Through Commissions and Kickbacks”. *American Economic Review* 102.2, pp. 780–809.

— (2012b). “Financial advice”. *Journal of Economic Literature* 50.2, pp. 494–512.

— (2012c). “How (not) to pay for advice: A framework for consumer financial protection”. *Journal of Financial Economics* 105.2, pp. 393–411.

Jeon, Doh-Shin and Nikrooz Nasr (2012). “News Aggregators and Competition among Newspapers in the Internet”. Working Paper.

Lazear, Edward P and Sherwin Rosen (1981). “Rank-Order Tournaments as Optimum Labor Contracts”. *The Journal of Political Economy* 89.5, pp. 841–864.

Lizzeri, Alessandro (1999). “Information Revelation and Certification Intermediaries”. *RAND Journal of Economics* 30.2, pp. 214–231.

Lu, Z. John and William S. Comanor (1998). “Strategic Pricing of New Pharmaceuticals”. *Review of Economics and Statistics* 80.1, pp. 108–118.

Meyer, Margaret A (1992). “Biased contests and moral hazard: Implications for career profiles”. *Annales d’Economie et de Statistique*, pp. 165–187.

Narasimhan, Chakravarthi (1988). “Competitive Promotional Strategies”. *Journal of Business* 61.4, pp. 427–449.

Peyrache, Eloic and Lucia Quesada (2011). “Intermediaries, credibility and incentives to collude”. *Journal of Economics & Management Strategy* 20.4, pp. 1099–1133.

Recode (2014). *Marissa Mayer’s Secret Plan to Get Apple to Dump Google and Default to Yahoo Mobile Search*. URL: `http://recode.net/2014/04/16/marissa-mayers-secret-plan-to-get-apple-to-dump-google-and-default-to-yahoo-mobile-search/`.

Rey, Patrick and Jean Tirole (2007). “A Primer on Foreclosure”. *Handbook of Industrial Organization, Volume 3*. Ed. by Mark Armstrong and Robert Porter. Amsterdam: Elsevier, pp. 2145–2220.

Rutt, James (2011). “Aggregators and the News Industry: Charging for Access to Content”. Working Paper.

Varian, Hal R. (1980). "A Model of Sales". *American Economic Review* 70.4, pp. 651–659.